www.mtg.in | January 2014 | ₹30

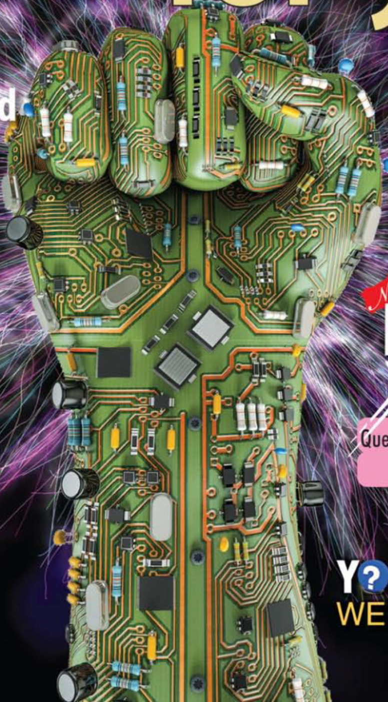
JEE FOUNDATION SERIES

Practice Paper AIPMT

Examiner's Mind
Class XI | Class XII

PHYSICS for you

India's #1
PHYSICS MONTHLY FOR JEE (Main & Advanced) & PMTs



CBSE Board
Sample Paper

PHYSICS
Musing

BRAIN
Map

THOUGHT PROVOKING PROBLEMS

New **NCERT Xtract**
Questions For Medical/Engg. Entrance Exams

YOU ASKED WE ANSWERED

MTG Learning Media
Trust of more than 1 Crore Readers Since 1982

PHYSICS for you

Vol. XXII No. 1 January 2014

Corporate Office:
Plot 99, Sector 44 Institutional area,
Gurgaon -122 003 (HR), Tel : 0124-4951200

Regd. Office
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
e-mail : info@mtg.in website : www.mtg.in

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat (BE, MBA)

Contents

Owned, Printed and Published by Mahabir Singh from 406, Taj Apartment, New Delhi - 29 and printed by Personal Graphics and Advertisers (P) Ltd., Okhla Industrial Area, Phase-II, New Delhi.

editOrial

Finding the Highest Common Factor between Music lovers and Multilingual children

It has been proved experimentally that scientists who love music are able to detect flaws in thinking much faster than others. Great scientists were also either great musicians or appreciated music very much. The correlation is very strong. Let us study a few examples. Helmholtz and Sir C.V. Raman had studied mathematically the various types of musical instruments and vocal music to find out the physics hiding in music. Fourier analysis came handy. The method remains the same today but totally computerized calculations are performed. The various fundamental frequencies and their overtones are studied like studying any superposition of waves of Fourier analysis.

Sir C.V. Raman had made exhaustive studies comparing many instruments to study the components of Ragas and percussion drums of various types. Studies came handy later to the Computer musicians to make synthesizers for many instruments. New combinations and frequency modulations were also tried by the Japanese and Chinese scientists – musicians. They were pioneers in this field.

Dr. Bhabha, Dr. Ramanna and Dr. P.N. Krishnamurthy were good musicians and music was encouraged in the laboratories as part of the improvement of the calibre of scientists. But what makes them tick? Any person who appreciates music can notice even small mistakes in the rendering of music. Every one of them can also extrapolate this ability to other activities such as scientific activities or mathematical research also. The principle of the transferability of skill is well accepted. This is also the reason why IAS and other cadres are changed from one department to another, including judiciary. Any person who can do very well in one department can pick up any other skill also very fast.

It has been reported recently that in U.S.A, Indians and other children who know three or four languages do far better than the children who know only English. This is the same as a musician knowing the various notes of a number of songs without confusing one song with another. Scientists and children are similar. Both look for beauty and order. But the scientist works in a single direction at a time knowing the purpose.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIPMT / AIIMS / Other PMTs / PETs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / AIPMT. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

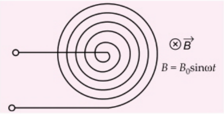
PROBLEM Set 6

SUBJECTIVE TYPE

1. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $\frac{R}{2}$.

2. A stone is projected from the point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height $2h$ above the ground. If at the instant of projection the bird were to fly away horizontally with uniform speed, find the ratio between horizontal velocities of the bird and stone if the stone still hits the bird while descending.

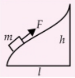
3. A plane spiral with a large number N of turns wound tightly to one another is located in a uniform magnetic field perpendicular to the plane of the spiral. The outside radius of the spiral's turns is equal to a. The magnetic induction varies with time as $B = B_0 \sin\omega t$, where B_0 and ω are constants. The amplitude of induced emf in the spiral is given by $\frac{\pi N a^2 B_0 \omega}{n}$. Find the value of n.



4. The pitch of a screw gauge is 1 mm and there are 50 divisions on its cap. When nothing is put in between the studs, the zero of the circular scale lies 6 division below the line of graduation. When a wire is placed between the studs, 3 linear scale divisions are clearly visible while 31 divisions on the circular scale coincide with the reference line. Determine the diametre of the wire.

SINGLE OPTION CORRECT

5. A body of mass m is slowly pulled up the hill by a force F which at each point was directed along the tangent of the trajectory as shown in figure. All surfaces are smooth. The work performed by this force is



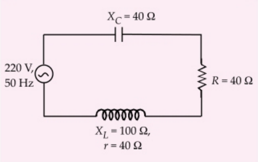
(a) mgl (b) $-mgl$
(c) mgh (d) zero

6. A loaded vertical spring executes simple harmonic oscillations with period of 4 s. The difference between the kinetic energy and potential energy of this system oscillates with a period of
(a) 8 s (b) 1 s
(c) 2 s (d) 4 s

7. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(a) $\frac{2R}{\sqrt{15}}$ (b) $R\sqrt{\frac{2}{15}}$ (c) $\frac{4R}{\sqrt{15}}$ (d) $\frac{R}{4}$

8. The power factor of the circuit shown in figure is



(a) 0.2 (b) 0.4 (c) 0.8 (d) 0.6

9. The current in an L-R circuit builds up to $\frac{3}{4}^{\text{th}}$ of its steady state value in 4 s. The time constant of this circuit is

(a) $\frac{1}{\ln 2}$ s (b) $\frac{2}{\ln 2}$ s

(c) $\frac{3}{\ln 2}$ s (d) $\frac{4}{\ln 2}$ s

10. From an atom of mass number 220, initially at rest, α-decay takes place. If the volume of the reaction is 5.5 MeV, the most probable K.E. of α-particle is

(a) 4.4 eV (b) 5.4 eV

(c) 5.6 eV (d) 6.5 eV.

EXAM ALERT

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY

ADMISSION TO B.TECH. PROGRAMME-2014

Indian Institute of Space Science and Technology (IIST), announces admission to its 4-year B. Tech. Degree Programmes for the academic year 2014-15, to eligible Indian citizens, in the following disciplines :

- Aerospace Engineering (60 seats)
- Avionics (60 seats)
- Physical Sciences (36 seats)

Candidates who are desirous to join B.Tech Programmes in IIST must appear for the Joint Entrance Examination (Main)-2014 conducted by Central Board of Secondary Education (CBSE), New Delhi to qualify themselves to appear for Joint Entrance Examination (Advanced)-2014 conducted by IITs. They must also appear for JEE (Advanced)-2014 and secure the minimum prescribed category-wise marks in JEE (Advanced)- 2014 as given below :

(a) General (GEN) Candidates : At least 20% marks in aggregate and at least 5% marks in each of the three subjects (Physics, Chemistry and Mathematics).

(b) Other Backward Community-Non Creamy Layer (OBC-NCL) Candidates : At least 18% marks in aggregate and at least 4.5% marks in each of the three subjects (Physics, Chemistry and Mathematics).

(c) Scheduled Caste (SC), Scheduled Tribe (ST) & Persons with Disabilities (PD) Candidates : At least 10% marks in aggregate and at least 2.5% marks in each of the three subjects (Physics, Chemistry and Mathematics).

More details of JEE (Main)-2014 may be viewed at **http://www.jeemain.nic.in**

More details of JEE (Advanced)-2014 may be viewed at **http:/www.jee.litd.ac.in**

For more details on the eligibility and procedures for B.Tech Admission in IIST, please visit **http://www.iist.ac.in/admission/under-graduate.**

IIST Admission Rank List : IIST Admission Rank list will be generated only for those candidates who register online for the admission to IIST and satisfying all the eligibility criteria. The IIST web portal for online registration will be opened in the month of May 2014. IIST Admission Rank list will be based on candidates' position in the All-India Rank list (including category rank) prepared and published by Central Board of Secondary Education (CBSE). CBSE prepares All-India rank list giving 60% weightage to the score in JEE (Main) examination and 40% weightage to the normalised score in Class 12th qualifying examination and is expected to be published in the first week of July, 2014.

(Please visit http://www.jeemain.nic.in for more details about the CBSE All-India rank list.)

Reservation of Seats : IIST follows the reservation policy of Government of India as applicable to Higher Educational Institutions for reservation of seats for various categories.

Financial Assistance : Each Semester, IIST provides Financial Assistance (funded by Department of Space, Government of India) for students who meet the performance standards set by IIST. This assistance covers the expenses of education viz. Tuition Fee, Students Amenity Fee, Hostel charges including dining, Establishment charges and Medical needs. In addition, a Book Allowance of Rs. 3000/- per semester is also provided to each student. (For more details, visit http://www.iist.ac.in/admission/under-graduate)

Absorption in ISRO/DOS : On successful completion of the B.Tech programme, the students who meet the specified academic standards set by IIST, may be absorbed in DOS/ISRO as Scientists/Engineers. As performance based financial assistance is provided by DOS to the B.Tech students of IIST, ISRO/DOS will have the first right to absorb them, subject to the human resource requirements of ISRO/DOS. The job absorption will be as per the recruitment norms of ISRO/DOS Centres for IIST graduates. (For more details on absorption in ISRO/DOS, visit http://www.iist.ac.in/admission/under-graduate)

Detailed information and IIST Admission Brochure shall be published by 30th April 2014.

For latest information please visit
http://www.iist.ac.in/admission/under-graduate
Contact email : admission2014@iist.ac.in

EXAMINER'S MIND

NCERT Class XI

The questions given in this column have been prepared strictly on the basis of NCERT Physics for Class XI. This year JEE (Main and Advanced)/NEET/AIIMS/other PMTs have drawn their papers heavily from NCERT books.

KINETIC THEORY, OSCILLATIONS AND WAVES

SECTION - 1

Only One Option Correct Type

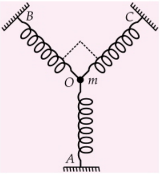
This section contains 15 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

1. How much should the pressure be increased in order to decrease the volume of a gas by 5% at a constant temperature?
   (a) 3.26% (b) 5.26%
   (c) 6.26% (d) 4.26%

2. A spring is loaded with two blocks m_1 and m_2, where m_1 is rigidly fixed with the spring and m_2 is just kept on the block m_1. The maximum energy of oscillation is possible for the system having the block m_2 in contact with m_1 is
   (a) $\frac{m_1^2 g^2}{k}$ (b) $\frac{m_1 g^2}{2k}$
   (c) $\frac{m_2^2 g^2}{2k}$ (d) $\frac{(m_1+m_2)^2 g^2}{2k}$

3. If y, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T. Then, which of the following does not change with time?
   (a) aT/y (b) $aT + 2\pi v$
   (c) aT/v (d) $a^2T^2 + 4\pi^2 v^2$

4. A particle of mass m is attached to three identical spring A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is



   (a) $2\pi\sqrt{\frac{2m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$
   (c) $2\pi\sqrt{\frac{m}{k}}$ (d) $2\pi\sqrt{\frac{m}{3k}}$

5. A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15°C. When the temperature falls to 10°C, the two produce 3 beats per second. Find the frequency (in Hz) of the fork.
   (a) 100 (b) 110 (c) 120 (d) 130

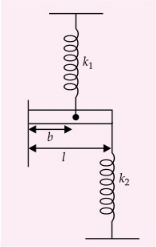
6. A pipe of length l_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in second chamber of gas of density ρ_2. The compressibility of both the gases is equal. The length of the second pipe if frequency of first overtone in both the cases is equal is
   (a) $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$
   (c) $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $l_1\sqrt{\frac{\rho_1}{\rho_2}}$

7. An object of specific gravity s is hung from a thin steel wire. The fundamental frequency of transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is
   (a) $300\sqrt{\left(1-\frac{1}{2s}\right)}$ (b) $300\sqrt{\left(\frac{2s}{2s-1}\right)}$
   (c) $300\left(\frac{2s}{2s-1}\right)$ (d) $300\left(\frac{2s-1}{2s}\right)$

8. A balloon contains 1500 m^3 of helium at 27°C and 4 atmospheric pressure. The volume of helium at –3°C temperature and 2 atmospheric pressure will be

(a) 1500 m^3 (b) 1700 m^3
(c) 1900 m^3 (d) 2700 m^3

9. A rod of mass m and length l is connected by two spring of spring constants k_1 and k_2, so that it is horizontal at equilibrium. What is the natural frequency of the system?



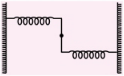
(a) $\frac{1}{2\pi}\sqrt{\frac{k_1 b^2 + k_2 l^2}{ml^2}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k_1 b^2 + k_2 l^2}{ml^2}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{k_1 b^2 + k_2 l^2}{2ml^2}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{3(k_1 b^2 + k_2 l^2)}{ml^2}}$

10. If three molecules have velocity 0.5 km s^{-1}, 1 km s^{-1}, and 2 km s^{-1} respectively. What is the ratio of the root mean square speed and average speed.
(a) 0.5 (b) 0.75
(c) 1.13 (d) 1.73

11. In Melde's experiment, a string vibrates in 3 loops when 8 g were placed in the pan. What mass must be placed in the pan to make the string vibrate in 5 loops?
(a) 2.88 g (b) 3.88 g (c) 4.88 g (d) 5.88 g

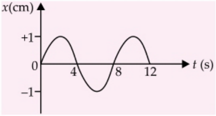
12. A uniform rod of length l and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid support as shown in figure and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{6M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

13. The x-t graph of a particle undergoing SHM is as shown in figure. The acceleration of the particle at $t = \frac{4}{3}$ s is

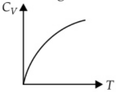


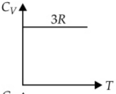
(a) $\frac{\sqrt{3}}{32}\pi^2$ cm s^{-2} (b) $-\frac{\pi^2}{32}$ cm s^{-2}

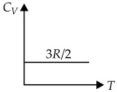
(c) $\frac{\pi^2}{32}$ cm s^{-2} (d) $-\frac{\sqrt{3}}{32}\pi^2$ cm s^{-2}

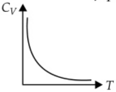
14. The length of second pendulum is 1 m on earth. If mass and diameter of the planet is doubled than that of earth, then length becomes
(a) 1 m (b) 2 m (c) 0.5 m (d) 4 m

15. Graph of specific heat at constant volume for a monoatomic gas is

(a) 

(b) 

(c) 

(d) 

SECTION - 2

One or More Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE are correct.

16. Three simple harmonic motions in the same direction having the same amplitude A and same period are suspended. If each differs in phase from the next by 45°, then
(a) the resultant amplitude is $(1+\sqrt{2})A$
(b) the phase of the resultant relative to the first is 90°
(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.
(d) the resultant motion is not simple harmonic.

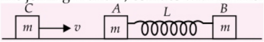
17. The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents simple harmonic motion for which of the option(s)?


BEST TOOLS FOR SUCCESS IN
JEE Main
READ
MTG
JEE MAIN
EXPLORER
₹275
MTG
12
2014
Years
FULLY REVISED EDITION
JEE main
CHAPTERWISE SOLUTIONS
PHYSICS | CHEMISTRY | MATHEMATICS
₹295
10
Very Similar
Practice Test Papers
12
JEE MAIN 2013
+
AIEEE (2012-2002)
Buy
online at
www.mtg.in
Available at all
leading bookshops
throughout India.
MTG
BOOKS
For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

(a) for all values of A, B and C ($C \neq 0$)
(b) $A = B$, $C = 2B$
(c) $A = -B$, $C = 2B$
(d) $A = B$, $C = 0$

18. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4}\sin(60t + 2x)$ where x and y are in metre and t is time in seconds. This represents a wave
(a) travelling with a velocity of 30 m s^{-1} in the negative x direction
(b) of wavelength π m
(c) of frequency $30/\pi$ Hz
(d) of amplitude 10^{-4} m travelling along the negative x-direction

19. Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in figure. A third identical block C, moving on the floor with a speed v along the line joining A and B, collides with A. Then



(a) The maximum compression of the spring is $v\sqrt{m/k}$
(b) The maximum compression of the spring is $v\sqrt{m/2k}$
(c) The K.E. of A-B system at maximum compression of the spring is zero.
(d) The K.E. of A-B system at maximum compression of the spring is $mv^2/4$.

20. The equation of SHM of a particle started at $t = 0$ is given by

$x = 5\sin(20t + \pi/3)$

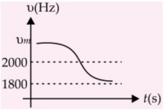
where x is in centimetre and t in second. Which of the following statements are correct?
(a) The particle first comes to rest at time $\dfrac{\pi}{120}$ s
(b) The particle first have zero acceleration at time $\dfrac{\pi}{30}$ s
(c) The particle first have maximum velocity at time $\dfrac{\pi}{30}$ s
(d) The particle first have maximum acceleration at time $\dfrac{\pi}{60}$ s

21. A wave disturbance in a medium is described by

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$$

where x and y are in metre and t is in second.
(a) A node occurs at $x = 0.15$.
(b) An a antinode occurs at $x = 0.3$ m.
(c) The speed of wave is 5 m s^{-1}.
(d) The wavelength is 0.2 m.

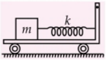
22. A stationary observer receives a sound of frequency $\upsilon_0 = 2000$ Hz. The apparent frequency υ varies with time as shown in figure. Speed of sound υ_0 is 300 m s^{-1}. Choose the correct alternative(s)



(a) Speed of source is 66.7 m s^{-1}
(b) υ_m shown in figure cannot be greater than 2500 Hz
(c) Speed of source is 33.33 m s^{-1}
(d) υ_m shown in figure cannot be greater than 2250 Hz

23. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is
(a) 25 kg (b) 5 kg
(c) 12.5 kg (d) $\dfrac{1}{25}$ kg

24. A block of mass m is attached to a massless spring of force constant k, the other end of which is fixed from the wall of a truck as shown in figure. The block is placed over a smooth surface and initially the spring is unstretched. Suddenly the truck starts moving towards right with a constant acceleration a_0. As seen from the truck



(a) the particle will execute SHM
(b) the time period of oscillations will be $2\pi\sqrt{\dfrac{m}{k}}$
(c) the amplitude of oscillations will be $\dfrac{ma_0}{k}$
(d) the energy of oscillations will be $\dfrac{m^2a_0^2}{k}$

25. A particle of mass m executes simple harmonic motion with amplitude A and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is

Crack BITSAT with ease

The BITSAT isn't a tough exam. It is only different from the others you will appear for. The book explores everything you need to crack BITSAT like a genius!

FREE CD INSIDE with

5 Model Test Papers

₹ 525

Complete Theory for

Mathematics, Physics, Chemistry
Logical Reasoning & English

Chapterwise MCQs for practice

1000+ MCQs in each section

5 Model Test Papers

Buy online at www.mtg.in

Available at leading bookshops throughout India.

MTG BOOKS

For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

(a) $2\pi^2 mA^2\upsilon^2$ (b) $\pi^2 mA^2\upsilon^2$

(c) $\frac{1}{4}mA^2\omega^2$ (d) $4\pi^2 mA^2\upsilon^2$

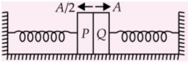
SECTION -3

Paragraph Type

This section contains 2 paragraphs each describing theory, experiment, data etc. Four questions related to two paragraphs with two questions on each paragraph. Each question of a paragraph has only one correct answer among the four choices (a), (b), (c) and (d).

Paragraph for Questions 26 and 27

Two identical blocks P and Q have mass m each. They are attached to two identical springs (of spring constant k) initially unstretched. Both the blocks are initially in contact as shown. Now the left spring (attached with block P) is compressed by $A/2$ and the right spring (attached with block Q) is compressed by A. Both the blocks are then released simultaneously.



Smooth horizontal floor

26. The speed of block Q just before P and Q are about to collide for the first time is

(a) $\sqrt{\frac{k}{m}}\frac{A}{2}$ (b) $\sqrt{\frac{k}{m}}A$

(c) $\sqrt{\frac{k}{2m}}A$ (d) None of these

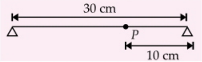
27. After what time when they were released from rest, shall the blocks collide for the first time?

(a) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ (b) $\pi\sqrt{\frac{m}{k}}$

(c) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (d) None of these

Paragraph for Questions 28 and 29

Figure shows a clamped metal string of length 30 cm and linear mass density 0.1 kg m^{-1}. Which is taut at a tension of 40 N. A small rider (piece of paper) is placed on string at point P as shown in figure. An external vibrating tuning fork is brought near the string and oscillations of rider are carefully observed.



28. At which of the following frequencies of tuning fork, rider will not vibrate at all

(a) $\frac{100}{3}$Hz (b) 50 Hz

(c) 200 Hz (d) None of these

29. At which of the following frequencies the point P on string will have maximum oscilliation amplitude among all points on string

(a) $\frac{200}{3}$Hz (b) 100 Hz

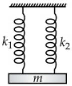
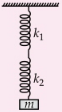
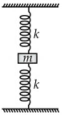
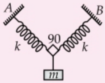
(c) 200 Hz (d) None of these

SECTION - 4

Matching List Type

This section contains 2 multiple choice questions. Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

30. Match List I with List II and select the correct answer using codes given below the Lists.

	List I		List II
P.		1.	$T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$
Q.		2.	$T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$
R.		3.	$T = 2\pi\sqrt{\frac{m}{k}}$
S		4.	$T = 2\pi\sqrt{\frac{m}{2k}}$

Codes

	P	Q	R	S
(a)	1	4	3	2
(b)	2	3	4	1
(c)	2	1	3	4
(d)	4	1	2	3

31. Match List I and List II and select the correct match out of the four given choices.

	List I		List II
P.	Tuning fork A of frequency 256 produces 4 beats per sec with fork B. Frequency of B may be	1.	128
Q.	A is loaded slightly. Beats frequency decreases Frequency of B is	2.	260
R.	A is filed slightly. Beats frequency decreases. Frequency of B is	3.	252
S.	If A were octave of B, frequency of B is	4.	264

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	3	4	1
(c)	4	3	2	1
(d)	1	3	2	4

SECTION - 5

Assertion-Reason Type

This section contains 4 questions. Read the two statements in the following questions. Of the four choices given, choose the one that best describes the two statements..

(a) Statement-I is true, Statement-II is true; Statement-I is a correct explanation of Statement-I.
(b) Statement-I is true, Statement-II is true; Statement-II is not a correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is false.
(d) Statement-I is false, Statement-II is true.

32. **Statement-I :** An open organ pipe of certain length have the same fundamental frequency as closed organ pipe of half the length.
Statement-II : In the case of open organ pipe, at both the ends antinodes are formed, while in the closed organ pipe at one end antinode and at the other end node is formed.

33. **Statement-I :** Compression and rarefraction involve changes in density and pressure.
Statement-II : When particles are compressed, density of medium increases and when they are rarefied, density of medium decreases.

34. **Statement-I :** When volume of a gas changes at constant temperature, root mean square velocity of its molecules remains unchanged.
Statement-II : The r.m.s. velocity of gas molecules does not depend upon volume of the gas.

35. **Statement-I :** Frequency of a particle executing SHM is 10 Hz. The particle is suspended from a verticle spring. At the highest point of its oscillation the spring is unstretched. Maximum speed of the particle is $\frac{1}{2\pi}$ m s^{-1}, when $g = 10$ m s^{-2}.
Statement-II : Maximum speed of particle,

$$v_{\max} = A\omega \text{ and } \omega = 2\pi\upsilon = \sqrt{\frac{k}{m}}$$

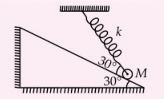
SECTION - 6

Integer Value Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

36. A particle is moving on x-axis has potential energy $U = (2 - 20x + 5x^2)$ J along x-axis. The particle is released at $x = -3$. If the mass of the particle is 10 kg, then find the angular frequency of the particle (in rad s^{-1}).

37. A sphere of mass M and radius R is on a smooth fixed inclined plane in equilibrium as shown in the figure. If now the sphere is displaced through a small distance along the plane, what will be angular frequency (in rad s^{-1}) of the resulting SHM? (Given $k = 4M/3$).



38. Speed of sound in air is 332 m s^{-1} at NTP. What will be the speed of sound (in km s^{-1}) in hydrogen at NTP if the density of hydrogen at NTP is (1/36) that of air?

39. A bus is moving with a velocity of 5 m s^{-1} towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air is 335 m s^{-1}, the number of beats heard per second by the passengers in the bus will be

40. A wire of length 128 cm produces a fundamental note of frequency 256 Hz, when stretched by a weight of 1 kg. By how much its length should be increased so that its frequency is resonates at 288 Hz, if it is now stretched by a weight of 4 kg?

SOLUTIONS

1. **(b):** Let initial pressure is P and initial volume is V. As the volume of gas is decreased by 5%,

$\therefore$ New volume, $V_1 = V - \dfrac{5}{100}V = \dfrac{95V}{100}$

Since at constant temperature

PV = constant

$\Rightarrow$ $PV = P_1V_1$

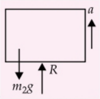
New pressure $P_1 = \dfrac{PV}{V_1} = \dfrac{PV}{(95/100)V} = \dfrac{100}{95}P$

$\therefore$ % increase in pressure

$$= \left(\frac{P_1 - P}{P}\right)\times 100 = \left(\frac{P_1}{P} - 1\right)\times 100$$

$$= \left(\frac{100}{95} - 1\right)\times 100 = 5.26\%$$

2. **(d):** Let x be the required amplitude of oscillation. For this amplitude of oscillation the normal contact force between the blocks can be given by



$R - m_2g = m_2a$

where, acceleration

$a = -\omega^2 x$

$\therefore$ $R - m_2g = -m_2\omega^2 x$

For maximum amplitude of oscillation without loosing contact of block m_2 with block m_1,

$R = 0.$

we get,

$x = g/\omega^2$

where ω is the angular frequency of oscillation of $(m_1 + m_2)$ and spring. Therefore

$$\omega = \sqrt{\frac{k}{(m_1+m_2)}} \text{ or } \omega^2 = \frac{k}{(m_1+m_2)}$$

$$\Rightarrow \quad x = \frac{(m_1+m_2)g}{k}$$

Hence, the maximum energy of oscillation

$$U_{max} = \frac{1}{2}kx^2 = \frac{(m_1+m_2)^2 g^2}{2k}$$

3. **(c) :** In S.H.M., displacement, $y = A\sin\omega t$

$$\therefore \quad v = \frac{dy}{dt} = A\omega\cos\omega t$$

$$\text{and } a = \frac{dv}{dt} = -A\omega^2\sin\omega t = -\omega^2 y$$

$$\text{So } \frac{aT}{y} = \frac{-\omega^2 y \times T}{y} = -\omega^2 T$$

$$= -\frac{4\pi^2}{T^2}\times T = -\frac{4\pi^2}{T} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

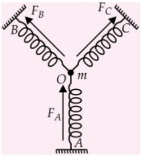
$$\text{and } \frac{aT}{v} = \frac{-\omega^2 A\sin\omega t \times T}{\omega A\sin\omega t}$$

$$= -\omega T = -\frac{2\pi}{T}\times T = -2\pi = \text{constant}$$

further $aT + 2\pi v$ and $a^2T^2 + 4\pi^2v^2$ are also not constant.

Hence, out of given options, $\dfrac{aT}{v}$ will not change with time.

4. **(b):** When the particle of mass m at O is pushed by y in the direction of A, the spring A will be compressed by y while springs B and C will be stretched by $y' = y\cos 45°$. So the total restoring force on the mass m along OA,



R.F. $= F_A + F_B\cos 45° + F_C\cos 45°$

$= ky + 2ky'\cos 45°$

$= ky + 2k(y\cos 45°)\cos 45° = k'y$

where $k' = 2k$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

5. **(b):** Let the frequency of the tuning fork be υ Hz.

Frequency of air column at 15°C = $\upsilon + 4$

Frequency of air column at 10°C = $\upsilon + 3$

According to $v = \upsilon\lambda$, we have

$v_{15} = (\upsilon + 4)\lambda$ and $v_{10} = (\upsilon + 3)\lambda$

$$\therefore \quad \frac{v_{15}}{v_{10}} = \frac{\upsilon+4}{\upsilon+3} \qquad \text{...(i)}$$

The speed of sound is directly proportional to the square root of the absolute temperature.

$$\therefore \quad \frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}} \qquad \text{...(ii)}$$

Equating (i) and (ii), we get

$$\therefore \quad \frac{\upsilon+4}{\upsilon+3} = \sqrt{\frac{288}{283}} = \left(1 + \frac{5}{283}\right)^{1/2}$$

$$\Rightarrow \quad 1 + \frac{1}{\upsilon+3} = 1 + \frac{1}{2}\times\frac{5}{283} = 1 + \frac{5}{566}$$

$$\Rightarrow \quad \frac{1}{\upsilon+3} = \frac{5}{566}$$

$$\Rightarrow \quad \upsilon + 3 = 113 \Rightarrow \upsilon = 110 \text{ Hz}$$

6. **(b):** Frequency of first overtone in closed pipe,

$$\upsilon = \frac{3v_1}{4l_1} = \frac{3}{4l_1}\sqrt{\frac{\gamma P}{\rho_1}} \quad \left[\because v = \sqrt{\frac{\gamma P}{\rho}}\right] \qquad \text{...(i)}$$

Frequency of first overtone in open pipe,

$$\upsilon' = \frac{2v_2}{2l_2} = \frac{1}{l_1}\sqrt{\frac{\gamma P}{\rho_2}} \qquad \text{...(ii)}$$

As $\upsilon = \upsilon'$

$\therefore$ From equations (i) and (ii), we get

$$\frac{3}{4l_1}\sqrt{\frac{\gamma P}{\rho_1}} = \frac{1}{l_2}\sqrt{\frac{\gamma P}{\rho_2}} \quad \Rightarrow \quad l_2 = \frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$$

7. **(a) :** The fundamental frequency is

$$\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

In air, $T = mg = V\rho g$ $[\because\ m = V\rho]$

$$\therefore \quad \upsilon = \frac{1}{2l}\sqrt{\frac{V\rho g}{\mu}} \quad \text{...(i)}$$

When the object is half immersed in water the apparent weight of object,

$$T' = mg - \text{upthrust}$$
$$= V\rho g - (V/2)\rho_w g = (V/2)g(2\rho - \rho_w)$$

The new fundamental frequency is

$$\upsilon' = \frac{1}{2l}\sqrt{\frac{T'}{\mu}} = \frac{1}{2l}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_w)}{\mu}} \quad \text{...(ii)}$$

Dividing (ii) by (i)

$$\frac{\upsilon'}{\upsilon} = \sqrt{\frac{2\rho-\rho_w}{2\rho}} \Rightarrow \upsilon' = 300\left(\sqrt{1-\frac{1}{2s}}\right)\text{Hz}$$

$\left[\because \text{ specific gravity } (s) = \text{relative density} = \dfrac{\text{density of object } (\rho)}{\text{density of water } (\rho_\omega)}\right]$

8. **(d):** $V_1 = 1500$ m3, $T_1 = 27°C = 300$ K
$P_1 = 4$ atm, $T_2 = -3°C = 270$ K
$P_2 = 2$ atm
According to ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

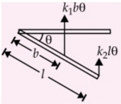
$$\therefore \quad V_2 = \frac{P_1V_1}{T_1}\times\frac{T_2}{P_2} = \frac{4\times1500\times270}{300\times2} = 2700 \text{ m}^3$$

9. **(d):** Let rod is displaced by an angle θ, taking torque about edge of rod

$$k_1b\theta \times b\cos\theta + k_2l\theta \times l\cos\theta = -\frac{Id^2\theta}{dt^2}$$

Here, $I = \dfrac{ml^2}{3}$ and θ is small $\cos\theta \approx 1$

$$\therefore \quad \frac{ml^2\alpha}{3} + (k_1b^2 + k_2l^2)\theta = 0$$



$$\Rightarrow \quad \alpha = \frac{-3(k_1b^2+k_2l^2)\theta}{ml^2}$$

$$\therefore \quad \omega = \sqrt{\frac{3(k_1b^2+k_2l^2)}{ml^2}}$$

and frequency $\upsilon = \dfrac{1}{2\pi}\sqrt{\dfrac{3(k_1b^2+k_2l^2)}{ml^2}}$

10. **(c) :** Root mean square speed,

$$v_{rms} = \sqrt{\frac{\sum v^2}{N}}$$

$$= \sqrt{\frac{(0.5)^2+(1)^2+(2)^2}{3}} \text{ km s}^{-1} = \sqrt{\frac{5.25}{3}} \text{ km s}^{-1}$$

$$= \sqrt{1.75} = 1.32 \text{ km s}^{-1}$$

Average speed,

$$\bar{v} = \frac{0.5+1+2}{3} \text{ km s}^{-1} = \frac{3.5}{3} \text{ km s}^{-1} = 1.17 \text{ km s}^{-1}$$

Thus the ratio, $\dfrac{v_{rms}}{\bar{v}} = \dfrac{1.32 \text{ km s}^{-1}}{1.17 \text{ km s}^{-1}} = 1.13$

11. **(a) :** Stretching force in string when,
$T_1 = m\rho = 8$ g wt
Number of loops, $n_1 = 3$
Let the stretching force be T_2,
When the number of loops $n_2 = 5$

As frequency of vibration $\upsilon = \dfrac{n}{2l}\sqrt{\dfrac{T}{\mu}}$

Case I. $\upsilon_1 = \dfrac{3}{2l}\sqrt{\dfrac{8 \text{ g wt}}{\mu}}$ ...(i)

Case II. $\upsilon_2 = \dfrac{5}{2l}\sqrt{\dfrac{T_2}{\mu}}$ ...(ii)

As in the both cases the frequency of vibration is same *i.e.*, $\upsilon_1 = \upsilon_2$

$$\therefore \quad \frac{3}{2l}\sqrt{\frac{8 \text{ g wt}}{\mu}} = \frac{5}{2l}\sqrt{\frac{T_2}{\mu}}$$

or $3\sqrt{8 \text{ g wt}} = 5\sqrt{T_2}$

Squaring both sides, we get

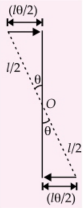
$9(8 \text{ g wt}) = 25T_2$

or $T_2 = \dfrac{9(8 \text{ g wt})}{25} = 2.88$ g wt

$\therefore$ mass = 2.88 g

12. **(c) :** If the rod is rotated through an angle θ, extension in one spring = compression in the other spring,

i.e., $x = l\theta/2$



Therefore, force acting on each of the ends of the rod,

$F = kx = k(l\theta/2)$

Restoring torque on the rod,

$\tau = -Fl = -k(l\theta/2)l = -kl^2(\theta/2)$

As $\tau = I\alpha = (Ml^2/12)\alpha$,

$-kl^2(\theta/2) = (Ml^2/12)\alpha$

or $\alpha = -\dfrac{6k}{M}\theta$, *i.e.*, $\alpha \propto \theta$

Thus, the motion of the rod is simple harmonic with

$$\omega^2 = \frac{6k}{M} \text{ or } \omega = \sqrt{\frac{6k}{M}}$$

or $\quad \upsilon = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

13. (d): The given curve is a sine curve with amplitude $A = 1$ cm and $T = 8$ s,

$$\omega = \frac{2\pi}{T} = \frac{\pi}{4}\ \text{s}^{-1}$$

If it is represented by $y = A\sin\omega t$, accelertation,

$$a = -\omega^2 A\sin\omega t = -(\pi/4)^2(1)\sin\left(\frac{\pi}{4}\times\frac{4}{3}\right)$$

$$\left[\text{Given } t = \frac{4}{3}\text{ s}\right]$$

$$= -\frac{\pi^2}{16}(\sqrt{3}/2) = -\frac{\sqrt{3}}{32}\pi^2\ \text{cm s}^{-2}$$

14. (c) : Time period of a second pendulum is 2 s.
When second pendulum is on earth, then

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad 2 = 2\pi\sqrt{\frac{R^2\times l}{GM}} \qquad \text{...(i)}$$

When second pendulum is on the planet, then

$$2 = 2\pi\sqrt{\frac{(2R)^2\times l'}{G\,(2M)}}$$

$$2 = 2\pi\sqrt{\frac{2R^2l'}{GM}} \qquad \text{...(ii)}$$

From (i) and (ii), we get $\sqrt{2l'} = l$

or $\quad l' = \frac{l^2}{2} = \frac{1}{2} = 0.5$ m

15. (c) : For a monotomic gas, $\gamma = \frac{5}{3}$

$\therefore$ Specific heat at constant volume

$$C_V = \frac{R}{\gamma - 1} = \frac{R}{\frac{5}{3}-1} = \frac{3}{2}R$$

Here, C_V is independent on T. Hence the graph will be a straight line of $C_V = \frac{3R}{2}$ parallel to T axis.

16. (a,c) : Using the principle of superposition,

$$\begin{aligned} y &= y_1 + y_2 + y_3 \\ &= A\sin(\omega t + 45°) + A\sin\omega t + A\sin(\omega t - 45°) \\ &= A[\sin(\omega t + 45°) + \sin(\omega t - 45°)] + A\sin\omega t] \\ &= 2A\sin\omega t\cos 45° + A\sin\omega t \\ &= \sqrt{2}\,A\sin\omega t + A\sin\omega t \end{aligned}$$

$$y = (1+\sqrt{2})A\sin\omega t$$

Amplitude of resultant motion = $(1+\sqrt{2})A$.
Hence option (a) is correct.
Option (b) is incorrect because the phase of resultant motion relative to the first is 45° not 90°.

$E \propto (\text{amplitude})^2 \qquad \left[\because E = \frac{1}{2}mA^2\omega^2\right]$

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \frac{(1+\sqrt{2})^2A^2}{A^2} = (3+2\sqrt{2})$$

$$= (3+2\sqrt{2})$$

$\Rightarrow \quad E_{\text{resultant}} = (3+2\sqrt{2})E$ single
Hence option (c) is correct
Option (d) is incorrect as the resultant motion is simple harmonic.

17. (a,b,c) : $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

or $\quad x = A\left(\frac{1-\cos 2\omega t}{2}\right) + B\left(\frac{1+\cos 2\omega t}{2}\right) + \frac{C\sin 2\omega t}{2}$

(a) For $A = 0$, $B = 0$, $x = \frac{C}{2}\sin 2\omega t$
This represents SHM. Hence option (a) is correct.

(b) For $A = B$, $C = 2B \Rightarrow x = B + B\sin 2\omega t$
This represents SHM of amplitude B. Hence option (b) is correct.

(c) For $A = -B$, $C = 2B$,
$x = B\cos 2\omega t + B\sin 2\omega t$
$= \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$
This represents SHM Hence option (c) is correct.

(d) For $A = B$, $C = 0$, $x = A$.
This does not represent SHM.

18. (a,b,c,d) : Here, $y = 10^{-4}\sin(60t + 2x)$
Comparing the given equation with the standard wave equation travelling in negative x-direction
$y = A\sin(\omega t + kx)$
We get amplitude, $A = 10^{-4}$ m
Also, $\omega = 60$ rad s^{-1}

$\therefore \quad 2\pi\upsilon = 60$

$\Rightarrow \quad \upsilon = \frac{30}{\pi}$ Hz

Also, $k = 2$

$\Rightarrow \quad \frac{2\pi}{\lambda} = 2 \Rightarrow \lambda = \pi$ m

Again the velocity of wave $v = \lambda\upsilon$

$$= \pi\times\frac{30}{\pi} = 30\ \text{m s}^{-1}$$

19. (b,d) : After collision of C with A, let velocity acquired by A and B be v' and spring gets compressed by length x. Using law of conservation of linear momentum, we have

$$mv = mv' + mv'$$

or $v' = v/2$

Using law of conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}kx^2$$

or $mv^2 = m\left(\frac{v}{2}\right)^2 + m\left(\frac{v}{2}\right)^2 + kx^2$

or $\frac{mv^2}{2} = kx^2$

or $x = v\left(\frac{m}{2k}\right)^{1/2}$

At maximum compression of the spring, the K.E. of A-B system will be

$$= \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 = mv^2 = m\left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}$$

20. (a,b,c) : Here, $x = 5\sin\left(20t + \frac{\pi}{3}\right)$.

$\therefore$ $A = 5$ cm; $\omega = 20$ rad s^{-1}

The particle will first come to rest when $x = 5$ cm

$\therefore$ $5 = 5\sin\left(20t + \frac{\pi}{3}\right)$

or $\sin\left(20t + \frac{\pi}{3}\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow$ $20t + \frac{\pi}{3} = \frac{\pi}{2}$ or $20t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

or $t = \frac{\pi}{120}$ s

The particle will have zero acceleration or maximum velocity while first passing through the mean position *i.e.* $x = 0$.

$\Rightarrow$ $0 = 5\sin\left(20t + \frac{\pi}{3}\right) = \sin\pi$

$\therefore$ $20t + \frac{\pi}{3} = \pi$ or $t = \frac{\pi}{30}$ s

The particle will have maximum acceleration at extreme position, where $x = 5$ cm, for which $t = \pi/120$ s.

21. (a,b,c,d) : Here,

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$$

Here, $A = 0.02$ m

$\omega = 50\pi$ rad s^{-1}, $k = 10\pi$ rad m^{-1}

$\therefore$ Velocity, $v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m s^{-1}

$\therefore$ Option (c) is correct.

Displacement node occurs at

$$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ etc. or } x = \frac{1}{20}, \frac{3}{20}$$

or $x = 0.05$ m, 0.15 m etc.

$\therefore$ Option (a) is correct.

Displacement antinode occurs at

$10\pi x = 0, \pi, 2\pi, 3\pi$ etc.

or $x = 0$, 0.1 m, 0.2 m, 0.3 m etc.

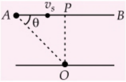
$\therefore$ Option (b) is correct

Wavelength, $\lambda = 2 \times$ distance between two consecutive noes or antinodes

$= 2 \times 0.1 = 0.2$ m

$\therefore$ option (d) is correct.

22. (c,d) : The graph shows the situation shown in figure. The observed frequency will initially be more than the natural frequency. When the source is at P, observed frequency is equal to its natural frequency *i.e.*, 2000 Hz.



For region AP, $\upsilon = \upsilon_o\left(\frac{v}{v - v_s\cos\theta}\right)$

For region PB, $\upsilon = \upsilon_o\left(\frac{v}{v + v_s\cos\theta}\right)$

Minimum value of ν will be

$$\upsilon_{\min} = \upsilon_o\left(\frac{v}{v + v_s}\right) \qquad \text{(when } \cos\theta = 1\text{)}$$

or $1800 = 2000\left(\frac{300}{300 + v_s}\right)$

Solving this we get, $v_s = 33.33$ m s^{-1}

and maximum value of ν can be

$$\upsilon_{\max} = \upsilon_o\left(\frac{v}{v - v_s}\right) \qquad \text{(when } \cos\theta = 1\text{)}$$

or $\upsilon_{\max} = 2000\left(\frac{300}{300 - 33.33}\right) = 2250$ Hz

23. (a) : The frequency of vibration of a string is given by

$$\upsilon = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

where p = number of loops (or segments)

= number of antinodes.

The other symbols have their usual meanings.

According to question,

$$\frac{5}{2L}\sqrt{\frac{9g}{\mu}}=\frac{3}{2L}\sqrt{\frac{Mg}{\mu}} \quad \text{or} \quad M=25\text{ kg}$$

24. (a,b,c)

25. (b,c) : For a particle to execute simple harmonic motion its displacement at any time t is given by

$y(t) = A\cos(\omega t + \phi)$

where, A = amplitude, ω = angular frequency,

ϕ = phase constant.

Let us choose $\phi = 0$

$\therefore \quad y(t) = a\cos\omega t$

Velocity of a particle $v=\frac{dx}{dt}=-a\omega\sin\omega t$

Kinetic energy of a particle is $K=\frac{1}{2}mv^2$

$$=\frac{1}{2}mA^2\omega^2\sin^2\omega t$$

Average kinetic energy $< K >$

$$=<\frac{1}{2}mA^2\omega^2\sin^2\omega t>$$

$$=\frac{1}{2}m\omega^2A^2<\sin^2\omega t>$$

$$=\frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right)=\frac{1}{4}m\omega^2A^2$$

$$\left[\because\ <\sin^2\theta>=\frac{1}{2}\right]$$

$$=\frac{1}{4}mA^2(2\pi\upsilon)^2 \qquad [\because\ \omega=2\pi\upsilon]$$

$$=\pi^2 mA^2\upsilon^2.$$

26. (b): As the right spring attached with block Q is compressed by A

Since $F = -kx$

$ma = -kx$

$a=\frac{-k}{m}x$

Comparing with standard relation

$\omega^2=\frac{k}{m}$

$\therefore \quad \omega=\sqrt{\frac{k}{m}}$

Here, speed of Q just before collision is

$$v_Q=\omega A=\sqrt{\frac{k}{m}}A$$

27. (a) : The blocks shall meet after time $t = T/4$, where T is time period of either isolated spring block system,

$$t=\frac{T}{4}=\frac{1}{4}2\pi\sqrt{\frac{m}{k}}=\frac{\pi}{2}\sqrt{\frac{m}{k}}$$

28. (c) : Wave velocity in string is

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{40}{0.1}}=20\text{ m s}^{-1}$$

Fundamental frequency of string oscillation is

$$\upsilon_0=\frac{v}{2l}=\frac{20}{0.6}=\frac{100}{3}\text{ Hz}$$

Thus string will be in resonance with a tuning fork of frequency,

$$\upsilon_f=\frac{100}{3}\text{Hz},\ \frac{200}{3}\text{Hz, 100Hz, }\frac{400}{3}\text{Hz, ...}$$

29. (d): Here rider will not oscillate at all only if it is at a node of stationary wave in all other cases of resonance and non-resonance. It will vibrate at the frequency of turning fork. At a distance $l/3$ from one end node will appear at 3rd, 6th, 9th or similar higher harmonics *i.e.* at frequencies 100 Hz, 200 Hz, ...

If string is divided in odd number of segments, these segments can never resonate simultaneously, hence at the location of rider, antinode is never obtained at any frequency.

30. (c) : P-2; Q-1; R-3; S-4

In figure (P), the effective spring constant,

$$K=k_1+k_2 \Rightarrow T=2\pi\sqrt{\frac{m}{K}}=2\pi\sqrt{\frac{m}{k_1+k_2}}$$

In figure (Q), the effective spring constant

$$\frac{1}{K}=\frac{1}{k_1}+\frac{1}{k_2}$$

or $\quad K=\frac{k_1k_2}{(k_1+k_2)}\Rightarrow 2\pi\sqrt{\frac{m}{K}}=2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

In figure (R), the effective spring constant,

$$K=k+k=2k \Rightarrow \quad T=2\pi\sqrt{\frac{m}{K}}=2\pi\sqrt{\frac{m}{2k}}$$

In figure (S), let the effective spring constant be K. If mass m be pulled down by distance y, the springs A and B will be stretched by

$y' = y\cos 45°$

Total restoring force is

$F = -Ky = F_A\cos 45° + F_B\cos 45° = (F_A + F_B)\cos 45°$

$= 2ky'\cos 45°$

$= 2k(y\cos 45°)\cos 45° = ky$

$\therefore \quad K = k$

$$T=2\pi\sqrt{\frac{m}{K}}$$

$$\therefore \quad T=2\pi\sqrt{\frac{m}{k}}$$

31. (b): P-2; Q-3; R-4; S-1

Possible frequencies of B are $256 \pm 4 = 260$ or 252

When A is loaded, its frequency decreases.

As beats frequency decreases, frequency of $B = 252$.

When A is field slightly, its frequency increases to 264. As beats frequency decreases, frequency of $B = 260$. When A is octave of B, frequency of $B = 256/2 = 128$.

32. (b) **33. (a)**

34. (c) : $v_{\text{rms}} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3P}{M/V}} = \sqrt{\dfrac{3PV}{M}}$

At constant temperature, PV = constant

$\therefore \quad v_{\text{rms}}$ = constant

35. (a) : Mean position of the particle is $\dfrac{mg}{k}$ distance below the unstretched position of spring. Amplitude of oscillation is

$$A = \frac{mg}{k}$$

and $\omega = \sqrt{\dfrac{k}{m}} = 2\pi\upsilon = 2\pi \times 10 = 20\pi$

or $\dfrac{m}{k} = \dfrac{1}{400\pi^2}$

Maximum speed of particle, $v_{\max} = A\omega = \left(\dfrac{mg}{k}\right)\omega$

$$= \left(\frac{g}{400\pi^2}\right)(20\pi) = \frac{1}{2\pi} \text{ m s}^{-1}$$

36. (1) : Here, $U = (2 - 20x + 5x^2)$

or $\qquad F = -\dfrac{dU}{dx} = 20 - 10x = -10(x-2)$

Hence, force constant $k = 10$; $m = 10$ kg

Angular frequency, $\omega = \sqrt{\dfrac{k}{m}} = 1 \text{ rad s}^{-1}$

37. (1) : The sphere is moved down through a distance x, then elongation in the spring $= x \cos 30°$

$\Rightarrow$ Restoring force $= k\,x \cos^2 30° = 3k\,x/4$

$$\Rightarrow \quad M\frac{d^2x}{dt^2} = \frac{-3}{4}kx \quad \Rightarrow \quad \frac{d^2x}{dt^2} = -\left(\frac{3k}{4M}\right)x$$

But $\dfrac{d^2x}{dt^2} = -\omega^2 x$

$$\therefore \quad \omega = \sqrt{\frac{3k}{4M}} = \sqrt{\frac{3}{4M} \times \frac{4}{3}M} = 1 \text{ rad s}^{-1}$$

$$\left(\because \quad k = \frac{4M}{3}\right)$$

38. (2) : The velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\therefore \quad \frac{v_H}{v_{\text{air}}} = \sqrt{\frac{P_H}{\rho_H} \times \frac{\rho_{\text{air}}}{P_{\text{air}}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_H}} \quad [\text{at NTP } P_{air} = P_H]$$

or $\quad v_H = v_{\text{air}} \times \sqrt{\dfrac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\dfrac{36}{1}} = 1992 \text{ m s}^{-1}$

$$\cong 2 \text{ km s}^{-1}$$

Thus speed of sound in hydrogen nearly equal to 2 k ms^{-1}

39. (5) : Given: $v_0 = 5 \text{ m s}^{-1}$, $\upsilon = 165$ Hz, $v = 335 \text{ m s}^{-1}$

$$\upsilon' = \frac{v + v_o}{v - v_s} \times \upsilon = \frac{335+5}{335-5} \times 165 = 170 \text{ Hz}$$

Therefore, number of beats heard per second

$= \upsilon - \upsilon' = 170 - 165 = 5$

40. (1) : In first case

$\upsilon = 256$ Hz, $l = 128$ cm, $T = 1 \times 1000 \times 980$ dyne

As $\quad \upsilon = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$

$$\therefore \quad 256 = \frac{1}{2 \times 128}\sqrt{\frac{1 \times 1000 \times 980}{\mu}} \qquad \text{...(i)}$$

In second case.

$\upsilon' = 288$ Hz, $T = 4 \times 1000 \times 980$ dyne

Let the length of the wire be increased by x cm. Its new length will be

$L' = L + x = (128 + x)$ cm

Now $\quad v' = \dfrac{1}{2L'}\sqrt{\dfrac{T}{\mu}}$

$$\text{or} \quad 288 = \frac{1}{2 \times (128 + x)}\sqrt{\frac{4 \times 1000 \times 980}{\mu}} \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\frac{256}{288} = \frac{128+x}{128} \times \frac{1}{\sqrt{4}}$$

$$\frac{8}{9} = \frac{128+x}{256}$$

On solving, $x = 100$ cm $= 1$ m

Solution Senders of Physics Musing

SET-5

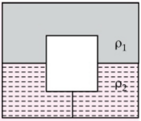
1. Nishita (Faridabad)
2. Vikram (Rajasthan)
3. Narendra Kumar (Karnataka)
4. Rohit Sharma (Delhi)
5. Ramesh Pandey (U.P.)

Thought Provoking Problems

FLUID MECHANICS

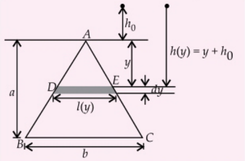
By : Prof. Rajinder Singh Randhawa*

1. A vessel contains two immiscible liquids of density $\rho_1 = 1000$ kg m^{-3} and $\rho_2 = 1500$ kg m^{-3}. A solid block of volume $V = 10^{-3}$ m^3 and density $d = 800$ kg m^{-3} is tied to

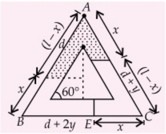


to one end of a string and other is tied to the bottom of the vessel as shown in figure. The block is immersed with $\frac{2}{5}^{\text{th}}$ of its volume in the liquid of higher density and $\frac{3}{5}^{\text{th}}$ in the liquid of lower density. The entire system is kept in an elevator which is moving upwards with an acceleration of $a = g/2$. Find the tension in the string. (Take $g = 10$ m s^{-2}).

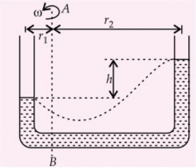
2. A triangular plate is submerged in a liquid of density ρ as shown in figure. What is the hydrostatic force on one face of the plate?



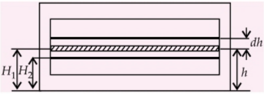
3. A glass tube in the form of an equilateral triangle of uniform cross-section is as shown in figure. It lies in the vertical plane, with base horizontal. The tube is filled with equal volumes of three immiscible liquids whose densities are in arithmetic progression. Determine the length x as shown in figure.



4. A U-tube shown in figure rotates with angular velocity ω about the vertical axis AB. What is the difference in fluid level h in terms of ω, the radii r and the fluid density ρ.



5. Consider a horizontal tank with an orifice of circular cross-section at the bottom. Assume R, radius of the tank; L, length of the tank ; H_1, initial height of the liquid; H_2, final height of the liquid. Determine the time taken to emtpy the tank from H_1 to H_2.

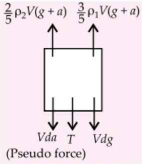


Randhawa Institute of Physics, S.C.O. 208, First Fl., Sector-36D & S.C.O. 38, Second Fl., Sector-20C, Chandigarh

6. A cylindrical vessel of area of cross-section A and filled with liquid to a height h_1 has a capillary tube of length l and radius r protruding horizontally at its bottom. If the viscosity of liquid is η, density ρ and $g = 9.8\ \text{m s}^{-2}$, find the time in which the level of water in the vessel falls to h_2.

SOLUTIONS

1. We analysed this problem from the reference frame of elevator.



Total buoyant force on the block,

$$F_B = \left(\frac{2}{5}V\rho_2 + \frac{3}{5}V\rho_1\right)(g+a)$$

For equilibrium,

$F_B = T + Vd(g+a)$

or $\quad T = F_B - Vd(g+a)$

$$= (g+a)V\left[\frac{2}{5}\rho_2 + \frac{3}{5}\rho_1 - d\right]$$

$$T = \left(10 + \frac{10}{2}\right)\times 10^{-3}\left[\frac{2}{5}\times 1500 + \frac{3}{5}\times 1000 - 800\right]$$

$= 6\ \text{N}$

2. Force on the element,

$dF = [\rho g(y + h_0)l(y)]\cdot dy$

where, $l(y)$ is the length of differential element at distance y from the top and dy its thickness.

$\therefore$ Total force is obtained as

$$F = \int dF = \int_0^a [\rho g(y+h_0)l(y)]dy \qquad \text{... (i)}$$

We use property of similar triangles on ΔABC and ΔADE, then

$$\frac{l(y)}{b} = \frac{y}{a}$$

$$\therefore \quad l(y) = \frac{b}{a}y \qquad \text{... (ii)}$$

Put (ii) in (i), we get

$$F = \int_0^a \rho g(y+h_0)\left(\frac{b}{a}y\right)dy = \rho g b\left[\frac{a^2}{3} + \frac{h_0 a}{2}\right]$$

3. Pressure at A,

Due to left arm, $P = dg(l-x)\sin 60° + (d+2y)gx\sin 60°$

Due to right arm, $P = dgx\sin 60° + (d+y)g\,(l-x)\sin 60°$

Equating the two values of pressure for equilibrium,

$$dg(l-x)\frac{\sqrt{3}}{2} + \frac{x\sqrt{3}}{2}(d+2y)g = \frac{x\sqrt{3}}{2}dg + (l-x)\frac{\sqrt{3}}{2}(d+y)g$$

or $\quad (l-x)d + x(d+2y) = xd + (l-x)(d+y)$

or $\quad ld - xd + xd + 2xy = xd + ld + ly - xd + xy$

$\Rightarrow \quad 2yx = y(l-x) \quad$ or $\quad 2x = l - x$

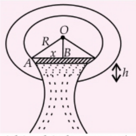
$$\therefore \quad x = \frac{l}{3}$$

4. Let y_{min} be the lowest point on the dotted parabola shown.

$$y_1 = \frac{r_1^2\omega^2}{2g} + y_{min} \quad\text{and}\quad y_2 = \frac{r_2^2\omega^2}{2g} + y_{min}$$

$$\therefore \quad h = y_2 - y_1 = \frac{(r_2^2 - r_1^2)\omega^2}{2g}$$

5. Consider a layer of thickness dh at a height h from base.



Volume of liquid in this layer $= -A\cdot dh$

$= -(2xL)dh \qquad$ ... (i)

Volume of liquid flowing through orifice in time dT,

$$= a\sqrt{2gh}\cdot dT \qquad \text{... (ii)}$$

$$\therefore \quad -2xL\cdot dh = a\sqrt{2gh}\cdot dT \;\Rightarrow\; dT = \frac{-2xL\cdot dh}{a\sqrt{2gh}}$$

From triangle OAB,

$$x = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$$

$$\therefore \quad dT = \frac{-2L\sqrt{(2R-h)}\cdot dh}{a\sqrt{2g}}$$

$$\therefore \quad T = \int dT = \frac{-2L}{a\sqrt{2g}}\int_{H_1}^{H_2}(2R-h)^{1/2}\cdot dh$$

$$T = \frac{4L}{3a\sqrt{2g}}[(2R-H_2)^{3/2} - (2R-H_1)^{3/2}]$$

6. Let h be the height of water level in the vessel at instant t which decreases by dh in time dt.

$\therefore$ Rate of flow of water through the capillary tube

$$V = -A\left(\frac{dh}{dt}\right) \qquad \text{... (i)}$$

Further, rate of flow from Poiseuille theorem,

$$V = \frac{\pi P r^4}{8\eta l} \qquad \text{... (ii)}$$

The hydrostatic pressure at depth h is $P = \rho g h$.

From eqn. (i) and (ii), we get

$$-A\frac{dh}{dt} = \frac{\pi\rho g h r^4}{8\eta l} \qquad \{\because P = \rho g h\}$$

$$\therefore \quad dt = \frac{-8\eta l A}{\pi\rho g r^4}\times\frac{dh}{h}$$

Integrating both sides, we get

$$t = \int dt = \frac{-8\eta l A}{\pi\rho g r^4}\int_{h_1}^{h_2}\frac{dh}{h} \;\Rightarrow\; t = \frac{8\eta l A}{\pi\rho g r^4}\log_e\frac{h_1}{h_2}$$

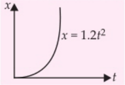
VERI- SIMILAR

PRACTICE PAPER 2014

Exam on 4th May

AIPMT Special

1. Figure shows the distance-time graph of the motion of a car. It follows from the graph that the car is



(a) at rest
(b) in uniform motion
(c) in non-uniform motion
(d) uniformly accelerated

2. A stone is thrown at an angle θ to the horizontal reaches a maximum height H. Then the time of flight of stone will be

(a) $\sqrt{\dfrac{2H}{g}}$ (b) $2\sqrt{\dfrac{2H}{g}}$

(c) $2\sqrt{\dfrac{2H\sin\theta}{g}}$ (d) $\sqrt{\dfrac{2H\sin\theta}{g}}$

3. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

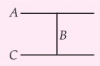
(a) K (b) zero (c) $\dfrac{K}{4}$ (d) $\dfrac{K}{2}$

4. If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the speed of light, then the values of p, q and r are, respectively

(a) $\dfrac{-1}{2}, \dfrac{1}{2}$ and $\dfrac{5}{2}$ (b) $\dfrac{1}{2}, \dfrac{-1}{2}$ and $\dfrac{-5}{2}$

(c) $\dfrac{-1}{2}, \dfrac{1}{2}$ and $\dfrac{3}{2}$ (d) $\dfrac{1}{2}, \dfrac{-1}{2}$ and $\dfrac{-3}{2}$

5. In figure, all three rods are of equal length L and same mass M. The system is rotated such that rod B is the axis of rotation. What is the moment of inertia of the system?



(a) $\dfrac{ML^2}{3}$ (b) $\dfrac{ML^2}{6}$ (c) $\dfrac{4}{3}ML^2$ (d) $\dfrac{2}{3}ML^2$

6. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\dfrac{n}{n+1}\right)mgR$ (b) $\left(\dfrac{n}{n-1}\right)mgR$

(c) $nmgR$ (d) $\dfrac{mgR}{n}$

7. Force acting on a particle is $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N, accelerates it with $|\vec{a}| = 1\text{ m s}^{-2}$. The mass of the particle is

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg
(c) 10 kg (d) 20 kg

8. A body floats in water with one-third of its volume above the surface of water. If it is placed in oil, it floats with half of its volume above the surface of the oil. The specific gravity of oil is

(a) $\dfrac{5}{3}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{3}{2}$ (d) 1

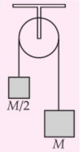
9. The excess of pressure inside the first soap bubble is three times that of inside the second bubble. The ratio of volume of the first to that of the second bubble is

(a) 1 : 3 (b) 1 : 9 (c) 1 : 27 (d) 9 : 1

10. Certain amount of gas is sealed in a glass flask at 1 atmospheric pressure and 20°C. The flask can withstand up to a pressure of 2 atmosphere. Find the temperature to which the gas can be heated so that the flask doesn't break.

(a) 513°C (b) 413°C (c) 313°C (d) 213°C

11. Two masses M and $M/2$ are joined together by means of light inextensible string passing over a frictionless pulley as shown in figure. When the bigger mass is released from equilibrium position, the smaller one will ascend with an acceleration of



(a) $\dfrac{g}{3}$ (b) $\dfrac{3g}{2}$ (c) g (d) $\dfrac{g}{2}$

12. The third overtone of an open organ pipe is in resonance with the second overtone of a closed organ pipe. If the length of the open pipe is 8 cm, then the length of the closed pipe is
(a) 10 cm (b) 8 cm (c) 12 cm (d) 5 cm

13. If Q, E and W denote respectively the heat added, change in internal energy and the work done in a closed cyclic process, then
(a) $W = 0$ (b) $Q = W = 0$
(c) $E = 0$ (d) $Q = 0$

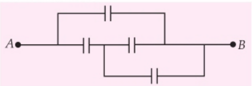
14. Two massless springs of force constants k_1 and k_2 are joined end to end. The resultant force constant K of the system is
(a) $\frac{k_1 + k_2}{k_1 k_2}$ (b) $\frac{k_1 - k_2}{k_1 k_2}$
(c) $\frac{k_1 k_2}{k_1 + k_2}$ (d) $\frac{k_1 k_2}{k_1 - k_2}$

15. A body constrained to move in the y direction is subjected to a force $\vec{F} = (2\hat{i} + 15\hat{j} + 6\hat{k})$ N. The work done by the force in moving the body through a distance of 10 m along y-axis is
(a) 100 J (b) 150 J (c) 60 J (d) 20 J

16. The equation $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$ where the symbols carry the usual meaning and A, T and λ are positive represents a wave of
(a) amplitude $2A$
(b) period $\frac{T}{\lambda}$
(c) speed $\frac{\lambda}{T}$
(d) velocity in negative x-direction

17. A particle is executing linear simple harmonic motion. The fraction of the total energy that is potential, when its displacement is $\frac{1}{2}$ of its amplitude is
(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

18. In the figure below, the capacitance of each capacitor is 3 μF. The effective capacitance between A and B is



(a) $\frac{3}{4}$ μF (b) 3 μF (c) 6 μF (d) 5 μF

19. Consider a car moving along a straight horizontal road with a speed of 72 km h^{-1}. If the coefficient of static friction between road and tyres is 0.5, the shortest distance in which the car can be stopped is (Take g = 10 m s^{-2})
(a) 30 m (b) 40 m
(c) 72 m (d) 20 m

20. An α particle is situated in an electric field of strength 15×10^4 N C^{-1}. Force acting on it is
(a) 4.8×10^{-12} N (b) 4.8×10^{-14} N
(c) 4.8×10^{-4} N (d) 4.8×10^{-10} N

21. A 2.0 cm tall object is placed 15 cm in front of a concave mirror of focal length 10 cm. What is the size and nature of the image?
(a) 4 cm, real (b) 4 cm, virtual
(c) 1.0 cm, real (d) none of these

22. Four point charges each $+q$ are placed on the circumference of a circle of diameter $2d$ in such a way that they form a square. The potential at the centre is
(a) 0 (b) $\frac{4q}{d}$ (c) $\frac{4d}{q}$ (d) $\frac{q}{4d}$

23. Half-life of a radioactive substance is 20 minute. The time between 20% and 80% decay will be
(a) 20 min (b) 30 min
(c) 40 min (d) 25 min

24. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 21%. The original charge on the capacitor is
(a) 10 C (b) 20 C (c) 30 C (d) 40 C

25. The instantaneous values of current and voltage in an AC circuit are given by
$I = 6\sin\left(100\pi t + \frac{\pi}{4}\right), V = 5\sin\left(100\pi t - \frac{\pi}{4}\right)$ then
(a) current leads the voltage by 45°
(b) voltage leads the current by 90°
(c) current leads the voltage by 90°
(d) voltage leads the current by 45°

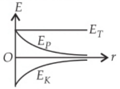
26. A force F acting on a body depends on its displacement S as $F \propto S^{-1/3}$. The power delivered by F will depend on displacement as
(a) $S^{2/3}$ (b) $S^{1/2}$
(c) S^0 (d) $S^{-5/3}$

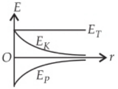
27. A car is moving in a circular horizontal track of radius 10 m with a constant speed 10 m s^{-1}. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1 m. The angle made by the rod with track is
(a) zero (b) 30° (c) 45° (d) 60°

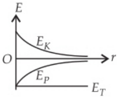
28. Centre of mass of three particles of masses 1 kg, 2 kg and 3 kg lies at the point (1, 2, 3) and centre of mass of another system of particles of 3 kg and 2 kg lies at the point (– 1, 3, – 2). Where should we put the particle of mass 5 kg so that the centre of masses of the entire system lies at the centre of mass of the first system?

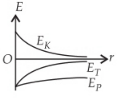
(a) (0, 0, 0) (b) (1, 3, 2)
(c) (– 1, 2, 3) (d) (3, 1, 8)

29. The correct graph representing the variation of total energy (E_T), kinetic energy (E_K) and potential energy (E_P) of a satellite with its distance (r) from the centre of earth is

(a) 

(b) 

(c) 

(d) 

30. If the value of acceleration due to gravity at the surface of a sphere of radius r is a_m, then its value will be $a_m/3$ at distance ..... from centre.

(a) $\sqrt{3}\,r$ (b) $r/\sqrt{3}$
(c) $2\sqrt{3}/r$ (d) $r/3$

31. A steel wire 1 m long and having 1 mm^2 cross sectional area is hung from a rigid support. When a weight of 1 kg is hung from it then change in length will be
(Given $Y = 2 \times 10^{11}$ N m^{-2})

(a) 0.5 mm (b) 0.25 mm
(c) 0.05 mm (d) 5 mm

32. A gas is compressed adiabatically till its temperature is doubled. The ratio of its final volume to initial volume will be

(a) 1/2 (b) more than 1/2
(c) less than 1/2 (d) between 1 and 2

33. In the two vessels of same volume atomic hydrogen and helium at pressure 1 atm and 2 atm are filled. If temperature of both the samples is the same, then average speed of hydrogen atoms $<c_H>$ will be related to that of helium $<c_{He}>$ as

(a) $<c_H> = \sqrt{2} <c_{He}>$ (b) $<c_H> = <c_{He}>$

(c) $<c_H> = 2 <c_{He}>$ (d) $<c_H> = \dfrac{<c_{He}>}{2}$

34. A cylinder of radius r and thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius r and outer radius $2r$ whose thermal conductivity is K_2. There is no loss of heat across cylindrical surfaces when the ends of the combined system are maintained at temperatures T_1 and T_2. The effective thermal conductivity of the system, under steady state condition is

(a) $\dfrac{(K_1K_2)}{(K_1+K_2)}$ (b) $(K_1 + K_2)$

(c) $\dfrac{(K_1+3K_2)}{4}$ (d) $\dfrac{(3K_1+K_2)}{4}$

35. A rod of length l and mass m is suspended horizontally in the middle by an inextensible string of length l and given torsional vibration. The time period of small oscillations is

(a) $\sqrt{\dfrac{l}{3g}}$ (b) $\sqrt{\dfrac{l}{12g}}$ (c) $\sqrt{\dfrac{m}{2g}}$ (d) $\sqrt{\dfrac{2l}{g}}$

36. A resonance tube is resonated with tuning fork of frequency 256 Hz. If the length of resonating air columns are 32 cm and 100 cm, then end correction will be

(a) 1 cm (b) 2 cm (c) 4 cm (d) 6 cm

37. $ABCD$ is a rectangle whose side AB = 10 cm and side BC = 24 cm. A charge of 0.104 μC is lying at the center O of rectangle. If the mid-point of side BC is E, then the work done in carrying 100 μC charge from B to E will be

(a) 1.152 J (b) 2.304 J
(c) 4.082 J (d) 23.4 J

38. An achromatic combination of lenses has the power of convex lens 2 D and ratio of dispersive powers of lenses are equal then the focal length of concave lens is

(a) –50 cm (b) 100 cm (c) 20 cm (d) 200 cm

39. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is

(a) $\dfrac{L}{l}$ (b) $\dfrac{L}{l}+1$ (c) $\dfrac{L}{l}-1$ (d) $\dfrac{L+1}{L-1}$

40. In a Young's double slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be

(a) $I_0 \cos\left(\dfrac{x}{\beta}\right)$ (b) $I_0 \cos^2\left(\dfrac{x}{\beta}\right)$

(c) $I_0 \cos^2\left(\dfrac{\pi x}{\beta}\right)$ (d) $\left(\dfrac{I_0}{4}\right)\cos^2\left(\dfrac{\pi x}{\beta}\right)$

41. A large solid sphere with uniformly distributed positive charge has a smooth narrow tunnel through its centre. A small particle with negative charge, initially at rest far from the sphere, approaches it along the line of the tunnel, reaches its surface with a speed v, and passes through the tunnel. Its speed at the centre of the sphere will be

(a) 0 (b) v (c) $\sqrt{2}v$ (d) $\sqrt{1.5}v$

42. A large flat metal surface has a uniform charge density $+\sigma$. An electron of mass m and charge e leaves the surface at point A with speed u, and returns to it at point B. Disregard gravity. The maximum value of AB is

(a) $\frac{u^2 m\varepsilon_0}{\sigma e}$ (b) $\frac{u^2 e\varepsilon_0}{m\sigma}$ (c) $\frac{u^2 e}{\varepsilon_0 \sigma m}$ (d) $\frac{u^2 \sigma e}{\varepsilon_0 m}$

43. A and B are two points on a uniform ring of resistance R. The $\angle ACB = \theta$, where C is the centre of the ring. The equivalent resistance between A and B is

(a) $\frac{R}{4\pi^2}(2\pi-\theta)\theta$ (b) $R\left(1-\frac{\theta}{2\pi}\right)$

(c) $R\frac{\theta}{2\pi}$ (d) $R\frac{2\pi-\theta}{4\pi}$

44. The magnetic field perpendicular to the plane of a conducting ring of radius r changes at the rate $\frac{dB}{dt}$.

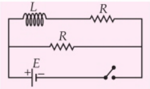
(a) The emf induced in the ring is $\pi r^2 \frac{dB}{dt}$.

(b) The emf induced in the ring is $2\pi r \frac{dB}{dt}$.

(c) The potential difference between diametrically opposite points on the ring is half of the induced emf.

(d) All points on the ring are at different potentials.

45. A coil of self inductance L and resistance R is connected to another resistance R and a cell of emf E, as shown in figure. The switch is kept closed for a long time and then opened. The heat produced in the coil, after opening the switch, is

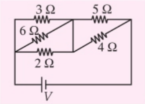


(a) $\frac{LE^2}{2R^2}$

(b) $\frac{LE^2}{4R^2}$

(c) $\frac{LE^2}{8R^2}$ (d) $\frac{2LE^2}{3R^2}$

46. The resistor in which maximum heat will be produced is



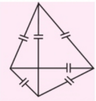
(a) 6 Ω

(b) 2 Ω

(c) 5 Ω

(d) 4 Ω

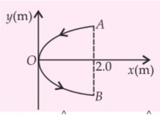
47. If the capacitance of each capacitor is C, then effective capacitance of the shown network across any two junctions is



(a) $2C$ (b) C

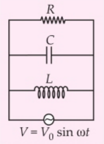
(c) $\frac{C}{2}$ (d) $5C$

48. A conducting wire bent in the form of the parabola $y^2 = 2x$ carries a current $I = 2$ A as shown in figure. This wire is placed in a uniform magnetic field $\vec{B} = -4\hat{k}$ T. The magnetic force on the wire is (in N)



(a) $-16\hat{i}$ (b) $32\hat{i}$ (c) $-32\hat{i}$ (d) $16\hat{i}$

49. In the circuit shown in figure $X_L = \frac{X_C}{2} = R$, the peak value current I_0 is



(a) $\frac{\sqrt{5}V_0}{2R}$ (b) $\frac{V_0}{2\sqrt{2}R}$

(c) $\frac{V_0}{2R}$ (d) $\frac{V_0}{2\sqrt{3}R}$

50. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the n^{th} orbit and its radius in the n^{th} orbit

(a) is proportional to n^2

(b) is inversely proportional to n^3

(c) has a constant value of 10.2 eV Å.

(d) has a constant value of 7.2 eV Å

SOLUTIONS

1. **(d):** $x = 1.2t^2$,

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(1.2t^2) = 2.4t$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(2.4t) = 2.4 =$ a constant

Thus the given motion is uniformly accelerated.

2. **(b):** Maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$

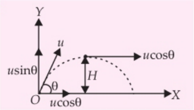
and time of flight, $T = \frac{2u\sin\theta}{g}$

where u is the velocity of the projection of the stone

$$\therefore \quad \frac{T^2}{H} = \left(\frac{2u\sin\theta}{g}\right)^2 \times \frac{2g}{u^2\sin^2\theta} = \frac{8}{g}$$

or $\quad T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$

3. **(c) :**



Here, angle of projection, $\theta = 60°$

Let u be the velocity of projection of the particle.

Kinetic energy of the particle at the point of projection O is

$$K = \frac{1}{2}mu^2 \qquad \text{...(i)}$$

where m is mass of the particle.

Velocity of the particle at the highest point (*i.e.* at maximum height) is $u\cos\theta$.

$\therefore$ Kinetic energy of the particle at the highest point is

$$K' = \frac{1}{2}m(u\cos\theta)^2$$

$$= \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}mu^2\cos^2 60°$$

$$= \frac{1}{2}mu^2\left(\frac{1}{2}\right)^2 = \frac{K}{4} \qquad \text{(Using (i))}$$

4. **(a) :** $E = G^p h^q c^r$ ...(i)

Equating dimensions on both sides of equation (i), we get

$[M^1L^2T^{-2}] = [M^{-1}L^3T^{-2}]^p[ML^2T^{-1}]^q[LT^{-1}]^r$

$= [M^{-p+q}\, L^{3p+2q+r}\, T^{-2p-q-r}]$

Applying principle of homogeneity of dimensions, we get

$-p + q = 1$ ...(ii)

$3p + 2q + r = 2$ ...(iii)

$-2p - q - r = -2$ ...(iv)

Adding (iii) and (iv), we get

$p + q = 0$ ...(v)

Adding (ii) and (v), we get

$$2q = 1 \text{ or } q = \frac{1}{2}$$

From (ii)

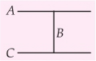
$$p = q - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

Substituting the values of p and q in equation (iii), we get

$$-\frac{3}{2} + 1 + r = 2 \text{ or } r = \frac{5}{2}$$

Hence, $p = -\frac{1}{2}, q = \frac{1}{2}, r = \frac{5}{2}$

5. **(b):** It is clear from figure, rod B passes through centre of rods A and C. Therefore, moment of inertia of the system about rod B is



$$I = I_A + I_B + I_C$$

$$I = \frac{ML^2}{12} + 0 + \frac{ML^2}{12} = \frac{ML^2}{6}$$

6. **(a) :** Gravitational potential energy of mass m at any point at a distance r from the centre of earth is

$$U = -\frac{GMm}{r}$$

At the surface of earth $r = R$,

$$\therefore \quad U_s = -\frac{GMm}{R} = -mgR \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

At the height $h = nR$ from the surface of earth

$r = R + h = R + nR = R(1 + n)$

$$\therefore \quad U_h = -\frac{GMm}{R(1+n)} = -\frac{mgR}{(1+n)}$$

Change in gravitational potential energy is

$$\Delta U = U_h - U_s = -\frac{mgR}{(1+n)} - (-mgR)$$

$$= -\frac{mgR}{1+n} + mgR$$

$$= mgR\left(1 - \frac{1}{1+n}\right) = mgR\left(\frac{n}{1+n}\right)$$

7. **(a) :** $|\vec{F}| = \sqrt{6^2 + (-8)^2 + (10)^2} = 10\sqrt{2}$ N

Mass of the particle,

$$m = \frac{|\vec{F}|}{|\vec{a}|} = \frac{10\sqrt{2}}{1} = 10\sqrt{2} \text{ kg}$$

8. **(b):** When the body is floating in a liquid, then

Weight of the body = Weight of liquid displaced

i.e. $V_{body}\,\rho_{body}\,g = V_{inside}\,\rho_{liquid}\,g$

Let V be the volume of the body.

In water

$$V\rho_{body}g = \frac{2}{3}V\rho_{water}g$$

$$\rho_{body} = \frac{2}{3}\rho_{water} \qquad \text{...(i)}$$

In oil

$$V\rho_{body}g = \frac{1}{2}V\rho_{oil}g$$

$$\rho_{body} = \frac{1}{2}\rho_{oil} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{\rho_{oil}}{\rho_{water}} = \frac{4}{3}$$

Specific gravity of oil, $= \frac{\rho_{oil}}{\rho_{water}} = \frac{4}{3}$

9. (c) : Excess of pressure inside the soap bubble,

$$P = \frac{4S}{R} \quad \text{or} \quad P \propto \frac{1}{R}$$

where, S is the surface tension and R is radius of soap bubble.

$$\therefore \quad \frac{P_1}{P_2} = \frac{R_2}{R_1} \quad \text{or} \quad \frac{3P_2}{P_2} = \frac{R_2}{R_1} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{1}{3}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

10. (c) : $P_1 = 1$ atm, $P_2 = 2$ atm
$T_1 = 20°\text{C} = 293$ K, $T_2 = ?$

As $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ (As volume remains constant)

$$\frac{1}{293} = \frac{2}{T_2} \quad \text{or} \quad T_2 = 586 \text{ K}$$

or $T_2 = (586 - 273)°\text{C} = 313°\text{C}$

11. (a) : Here, $m_1 = M, m_2 = \frac{M}{2}$

The common acceleration of the system is

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{\left(M - \frac{M}{2}\right)g}{M + \frac{M}{2}} = \frac{g}{3}$$

12. (d): In an open pipe, the fundamental frequency is $\frac{v}{2L_O}$ and all harmonics are present. Third overtone means fourth harmonic.

$\therefore$ Frequency of third overtone, $\upsilon_4 = \frac{4v}{2L_O}$

where v is the velocity of sound and L_O is the length of the open pipe.

In a closed pipe, the fundamental frequency is $\frac{v}{4L_C}$ and only odd harmonics are present. Second overtone means fifth harmonic.

$\therefore$ Frequency of second overtone, $\upsilon_5 = \frac{5v}{4L_C}$

where L_C is the length of the closed pipe.

As $\upsilon_4 = \upsilon_5$

$$\frac{4v}{2L_O} = \frac{5v}{4L_C} \quad \text{or} \quad L_C = \frac{10}{16}L_O = \frac{10}{16} \times 8 \text{ cm} = 5 \text{ cm}$$

13. (c) : In a closed cyclic process, the system returns to its initial state. Therefore, the change in internal energy is zero, *i.e.*, $E = 0$.

14. (c) : Here, the springs are connected in series, their resultant force constant K is given by

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2} \quad \text{or} \quad K = \frac{k_1 k_2}{k_1 + k_2}$$

15. (b): Here, $\vec{F} = (2\hat{i} + 15\hat{j} + 6\hat{k})$ N and $\vec{r} = 10\hat{j}$ m

$$\therefore \quad W = \vec{F} \cdot \vec{r} = (2\hat{i} + 15\hat{j} + 6\hat{k}) \cdot 10\hat{j} = 150 \text{ J}$$

16. (c) : $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

Speed of wave $= \frac{\lambda}{T}$

Amplitude $= A$, Period $= T$,
Velocity is along $+x$-direction.

17. (d): In SHM,

Potential energy of the particle, $U = \frac{1}{2}m\omega^2 x^2$

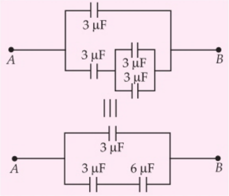
Total energy of the particle, $E = \frac{1}{2}m\omega^2 A^2$

where symbols have their usual meaning.

At $x = \frac{A}{2}$, the fraction of total energy which is potential energy

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 x^2}{\frac{1}{2}m\omega^2 A^2} = \frac{x^2}{A^2} = \left[\frac{A/2}{A}\right]^2 = \frac{1}{4} \quad \left[\because x = \frac{A}{2}\right]$$

18. (d): The equivalent circuit diagram is as shown in the figure.



The effective capacitance between A and B is

$$C_{AB} = 3\,\mu F + \frac{3\,\mu F \times 6\,\mu F}{3\,\mu F + 6\,\mu F} = 3\,\mu F + 2\,\mu F = 5\,\mu F$$

19. (b): Here, $v = 72$ km h^{-1} = 20 m s^{-1}

$\mu_s = 0.5$

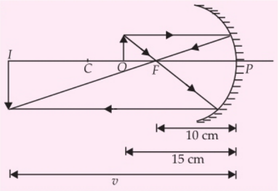
$$\mu_s mg\, d = \frac{1}{2} mv^2$$

or $$d = \frac{v^2}{2\mu_s g} = \frac{20 \times 20}{2 \times 0.5 \times 10} = 40 \text{ m}$$

20. (b): $F = qE = (2e)E$

$= (2 \times 1.6 \times 10^{-19}) \times 15 \times 10^4 = 4.8 \times 10^{-14}$ N

21. (a) :



According to new cartesian sign convention,
Object distance, $u = -15$ cm
Focal length of the concave mirror, $f = -10$ cm
Height of the object, $h_O = 2'$ cm

According to mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-15} = \frac{1}{-10} + \frac{1}{15}$$

or $v = -30$ cm.

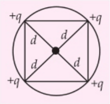
The image is formed at a distance of 30 cm from the mirror on the same side of the object. It is a real image.

Magnification of the mirror, $m = \frac{-v}{u} = \frac{h_I}{h_O}$

$$\Rightarrow \frac{-(-30)}{-15} = \frac{h_I}{2} \quad \Rightarrow \quad h_I = -4 \text{ cm.}$$

Negative sign shows that image is inverted.
The image is real, inverted, of size 4 cm at a distance of 30 cm in front of the mirror.

22. (b): Potential at centre due to all charges is



$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{d} + \frac{q}{d} + \frac{q}{d} + \frac{q}{d}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4q}{d} \text{ in S.I. units}$$

$$= \frac{4q}{d} \text{ in C.G.S. units}$$

23. (c) : According to radioactive decay,

$N = N_0 e^{-\lambda t}$

where,
N_0 = Number of radioactive nuclei present in the sample at $t = 0$
N = Number of radioactive nuclei left undecayed after time t
λ = decay constant.

For 20% decay

$$\frac{80N_0}{100} = N_0 e^{-\lambda t_1} \quad \text{...(i)}$$

For 80% decay

$$\frac{20N_0}{100} = N_0 e^{-\lambda t_2} \quad \text{...(ii)}$$

Dividing equation (i) by (ii), we get

$4 = e^{-\lambda(t_1 - t_2)}$

$\Rightarrow \quad 4 = e^{\lambda(t_2 - t_1)}$

Taking natural logarithms of both sides of above equation, we get

$\ln 4 = \lambda(t_2 - t_1)$

or $$2 \ln 2 = \frac{\ln 2}{T_{1/2}}(t_2 - t_1)$$

$\Rightarrow \quad t_2 - t_1 = 2 \times T_{1/2} = 2 \times 20$ min
$= 40$ min

24. (b): Energy stored in the capacitor

$$U = \frac{1}{2}\frac{Q^2}{C} \quad \text{...(i)}$$

Here, $U' = U + 21\%$ of U

$$= U + \frac{21U}{100} = 1.21U$$

$$\therefore \quad 1.21U = \frac{1}{2}\frac{(Q+2)^2}{C} \quad \text{...(ii)}$$

Divide (ii) by (i) we get

$$\frac{1.21}{1} = \frac{(Q+2)^2}{Q^2} \text{ or } \sqrt{\frac{1.21}{1}} = \frac{Q+2}{Q}$$

or $1.1Q = Q + 2$ or $(1.1 - 1)Q = 2$

or $0.1Q = 2$ or $Q = \frac{2}{(1/10)} = 20$ C

25. (c) : Here, $I = 6\sin\left(100\pi t + \frac{\pi}{4}\right)$

$$V = 5\sin\left(100\pi t - \frac{\pi}{4}\right)$$

Phase difference between current and voltage

$$= \left(100\pi t + \frac{\pi}{4}\right) - \left(100\pi t - \frac{\pi}{4}\right) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} = 90°$$

Hence, current leads the voltage by 90°

26. (c) : As $F \propto S^{-1/3}$, therefore, acceleration, $a \propto S^{-1/3}$

$$a = \frac{dv}{dt} = \frac{dv}{dS}\cdot\frac{dS}{dt} = \frac{dv}{dS}v \quad i.e. \quad v\frac{dv}{dS} \propto S^{-1/3}$$

Integrating both sides, we get

$v^2 \propto S^{2/3}$ or $v \propto S^{1/3}$

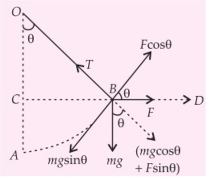
As $P = Fv$

$\therefore$ $P \propto S^{-1/3}\, S^{1/3}$ or $P \propto S^0$.

i.e. Power is independent of S.

27. (c) : Centrifugal force on rod, $F = \frac{mv^2}{r}$ along BD.

Let θ be the angle which the rod makes with the vertical. Forces acting on the rod are shown in figure.



Resolving mg and F into two rectangular components, we have forces parallel to rod,

$$mg\cos\theta + \frac{mv^2}{r}\sin\theta = T$$

Forces perpendicular to rod

$$= mg\sin\theta - \frac{mv^2}{r}\cos\theta.$$

The rod will be balanced if

$$mg\sin\theta - \frac{mv^2}{r}\cos\theta = 0 \quad \text{or} \quad mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

or $\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{10\times10} = 1 = \tan 45°$ or $\theta = 45°$

28. (d): According to the definition of centre of mass, we can imagine one particle of mass (1 + 2 + 3) kg at (1, 2, 3) and another particle of mass (2 + 3) kg at (–1, 3, – 2).

Let the third particle of mass 5 kg be put at (x_3, y_3, z_3) *i.e.*

$m_1 = 6$ kg, $(x_1, y_1, z_1) = (1, 2, 3)$

$m_2 = 5$ kg, $(x_2, y_2, z_2) = (-1, 3, -2)$

$m_3 = 5$ kg, $(x_3, y_3, z_3) = ?$

Given $(x_{cm}, y_{cm}, z_{cm}) = (1, 2, 3)$

Using $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$1 = \frac{6\times1 + 5\times(-1) + 5x_3}{6+5+5}$$

$5x_3 = 16 - 1 = 15$ or $x_3 = 3$.

Similarly, $y_3 = 1$ and $z_3 = 8$.

29. (d): $E_T = -\frac{GMm}{2r}$;

Thus E_T increases with increase in r.

$E_K = \frac{GMm}{2r}$; Thus E_K increases with increase in r.

$E_P = \frac{-GMm}{r}$; Thus E_P decreases with increase in r.

30. (d): Let y be the distance from the centre of sphere where the acceleration due to gravity is $a_m/3$

$\therefore y = r - d$

where r is the radius of the sphere and d is the depth from the surface of the sphere.

As $g' = g\left(1 - \frac{d}{r}\right) = g\left(\frac{r-d}{r}\right)$

$\therefore \quad \frac{a_m}{3} = a_m\left(\frac{y}{r}\right)$ or $y = \frac{r}{3}$

31. (c) : $Y = \frac{Fl}{A\Delta l}$

$$2\times10^{11} = \frac{1\times10}{10^{-6}}\times\frac{1}{\Delta l} \qquad (\because F = mg)$$

$$\Delta l = \frac{10}{2\times10^5} = 5\times10^{-5}\,\text{m}$$

$= 0.05 \times 10^{-3} = 0.05$ mm

32. (c) : For adiabatic process,

PV^γ = constant

Since $PV = RT$, hence $TV^{\gamma-1}$ = constant

$$\therefore \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right) = \frac{1}{2}$$

Hence $\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} < \left(\frac{1}{2}\right)$

33. (c) : Average speed $\langle c \rangle = \sqrt{\frac{8RT}{\pi M}} = 1.6\sqrt{\frac{RT}{M}}$

$\therefore \quad \langle c_H \rangle = 1.6\sqrt{\frac{RT}{1}}$

$\therefore \quad \langle c_{He} \rangle = 1.6\sqrt{\frac{RT}{4}}$

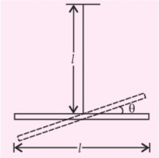
$\because$ temperature of both the samples is same.

$\Rightarrow \langle c_H \rangle = 2\langle c_{He} \rangle$

34. (c) : $\frac{K_1\pi r^2\Delta Tt}{l} + K_2\frac{\pi\left[(2r)^2 - r^2\right]\Delta Tt}{l} = \frac{K\pi(2r)^2\Delta Tt}{l}$

$\therefore \quad K = \frac{K_1 + 3K_2}{4}$

35. (b) : $T = 2\pi\sqrt{\frac{I}{mgl}} = \sqrt{\frac{ml^2/12}{mgl}} = \sqrt{\frac{l}{12g}}$



36. (b): $\frac{\lambda}{4} = 32 + 0.3d$ ...(i)

[d = diameter of the tube]

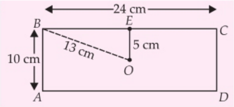
$\frac{3\lambda}{4} = 100 + 0.3d$ ...(ii)

From (i) and (ii), we get

$\frac{\lambda}{2} = 68$ cm

$\frac{\lambda}{4} = 34$ cm or $0.3d = 2$ cm.

37. (a):



$$W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

$$= 100 \times 0.104 \times 10^{-12} \times 9 \times 10^9 \left[\frac{1}{0.05} - \frac{1}{0.13}\right]$$

$$= 1.04 \times \frac{72}{65} = 1.152 \text{ J}$$

38. (a): For achromatic combination

$\frac{\omega_1}{f_1} = \frac{-\omega_2}{f_2}$ or $\omega_1 P_1 = -\omega_2 P_2$

or $P_2 = -P_1 = -2$ D ($\because \omega_1 = \omega_2$)

Focal length $= \frac{1}{P_2} = \frac{-1}{2} = -0.5$ m

or focal length = – 50 cm.

39. (a): Let f_o and f_e be the focal lengths of the objective and eyepiece respectively. For normal adjustment, distance between the objective and the eyepiece (tube length) = $f_o + f_e$. Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$$\frac{1}{v} - \frac{1}{-(f_o + f_e)} = \frac{1}{f_e}$$

or $\frac{1}{v} = \frac{1}{f_e} - \frac{1}{f_o + f_e} = \frac{f_o}{(f_o + f_e)f_e}$

or $v = \frac{(f_o + f_e)f_e}{f_o}$.

Magnification $= \left|\frac{v}{u}\right| = \frac{f_e}{f_o} = \frac{\text{image size}}{\text{object size}} = \frac{l}{L}$.

$\therefore \frac{f_o}{f_e} = \frac{L}{l}$ = magnification of telescope in normal adjustment.

40. (c): Path difference, $\Delta = x\frac{d}{D}$

and phase difference, $\phi = \frac{2\pi}{\lambda}\Delta$

Let a = amplitude at the screen due to each slit.

$\therefore I_0 = k(2a)^2 = 4ka^2$, where k is a constant.

For phase difference ϕ, amplitude

$= A = 2a\cos(\phi/2)$.

Intensity,

$$I = kA^2 = k(4a^2)\cos^2(\phi/2) = I_0\cos^2\left(\frac{\pi}{\lambda}\Delta\right)$$

$$I = I_0\cos^2\left(\frac{\pi}{\lambda}.\frac{xd}{D}\right) = I_0\cos^2\left(\frac{\pi x}{\beta}\right) \quad \left[\because \beta = \frac{\lambda D}{d}\right]$$

41. (d): Potential at $\infty = V_\infty = 0$.

Potential at the surface of the sphere, $V_s = k\frac{Q}{R}$.

Potential at the centre of the sphere, $V_c = \frac{3}{2}k\frac{Q}{R}$.

Let m and $-q$ be the mass and the charge of the particle respectively.

Let v_0 = speed of the particle at the centre of the sphere.

$$\frac{1}{2}mv^2 = -q\left[V_\infty - V_s\right] = qk\frac{Q}{R}.$$

$$\frac{1}{2}mv_0^2 = -q\left[V_s - V_c\right] = q\cdot\frac{3}{2}k\frac{Q}{R}.$$

Dividing, $\frac{v_0^2}{v^2} = \frac{3}{2} = 1.5$ or $v_0 = \sqrt{1.5}v$.

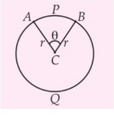
42. (a): The force on the electron is $\frac{e\sigma}{\varepsilon_0}$ and its acceleration towards the metal sheet is $\frac{e\sigma}{m\varepsilon_0}$.

The electron will move as the projectile with an effective value of $g = e\sigma/m\varepsilon_0$

Its maximum range will then be $\frac{u^2}{e\sigma/m\varepsilon_o} = \frac{u^2 m\varepsilon_0}{e\sigma}$

43. (a): Resistance per unit length $= \rho = \frac{R}{2\pi r}$.

Lengths of sections APB and AQB are $r\theta$ and $r(2\pi - \theta)$ respectively.



Resistance of section APB and AQB are

$$R_1 = \frac{R}{2\pi r}r\theta = \frac{R\theta}{2\pi}$$

and $R_2 = \frac{R}{2\pi r} r(2\pi - \theta) = \frac{R(2\pi - \theta)}{2\pi}$.

As R_1 and R_2 are in parallel between A and B,

$$\therefore \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R}{E\pi^2}(2\pi - \theta)\theta$$

44. (a) : $\phi = \pi r^2 B$

$$\varepsilon = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt}$$

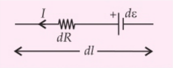
Let R = resistance of the ring

$\therefore$ Current in the ring = $i = \varepsilon/R$.

Consider a small element dl on the ring.

Emf induced in the element = $d\varepsilon = \left(\frac{\varepsilon}{2\pi r}\right)dl$

Resistance of the element $dR = \left(\frac{R}{2\pi r}\right)dl$



p.d. across the element

$$= -IdR + de = -\left(\frac{\varepsilon}{R}\right)\left(\frac{R}{2\pi r}\right)dl + \left(\frac{\varepsilon}{2\pi r}\right)dl = 0.$$

$\therefore$ all points on the ring are at the same potential.

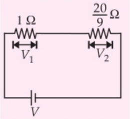
45. (b): In the steady state, the current in the coil is $I = \frac{E}{R}$. The energy stored in it is $= \frac{1}{2}LI^2 = \frac{LE^2}{2R^2}$.

When the switch is opened, this energy is shared equally between the two resistances.

46. (d): 3 Ω, 6 Ω and 2 Ω resistances are in parallel. So, potential drop across them will be equal. Of these three resistances maximum heat will be generated across 2 Ω resistance

$$\left(\because \ H = \frac{V^2}{R}t \text{ or } H \propto \frac{1}{R}\right)$$

Similarly, 5 Ω ancd 4 Ω resistances are also in parallel so, more heat will be generated across 4 Ω resistor. Now the given circuit can be redrawn as :



$$\frac{V_1}{V_2} = \frac{1}{\frac{20}{9}} = \frac{9}{20}$$

$$\therefore \quad V_1 = \left(\frac{9}{29}\right)V$$

and $V_2 = \left(\frac{20}{29}\right)V$

Power developed across 2 Ω resistor will be

$$P_1 = \frac{V_1^2}{2} = \left(\frac{9}{29}\right)^2 \frac{V}{2}$$

Power developed across 4 Ω resistor,

$$P_2 = \left(\frac{20}{29}\right)^2 \frac{V}{4} \quad \therefore \ P_2 > P_1.$$

47. (a) : The given network is a balanced wheatstone bridge with one capacitor in parallel with this bridge.

48. (b): $\vec{F}_{AOB} = \vec{F}_{AB} = I(\vec{l} \times \vec{B})$

Here, $AB = 2\sqrt{2 \times 2} = 4$ m

$$\therefore \quad \vec{F}_{AB} = 2[(-4\hat{j}) \times (-4\hat{k})] = 32\hat{i}$$

49. (a) : $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$

Substituting the given values, we get

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{2R} - \frac{1}{R}\right)^2}$$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{4R^2}}$$

$$\frac{1}{Z} = \frac{\sqrt{5}}{2R}$$

or $Z = \frac{2R}{\sqrt{5}} \quad \therefore \ I_0 = \frac{V_0}{Z} = \frac{\sqrt{5}V_0}{2R}$

50. (d): $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$

$\therefore E_n r_n$ is independent of n

Hence, $E_1 r_1 = (13.6 \text{ eV})(0.53 \text{ Å})$

$= 7.2$ eV Å = constant

NCERT Xtract

New

Questions for Medical/ Engineering Entrance Exams

Electric Charges and Fields

1. An electron initially at rest falls a distance of 1.5 cm in a uniform electric field of magnitude 2×10^4 N C^{-1}. The time taken by the electron to fall this distance is

(a) 1.3×10^2 s (b) 2.1×10^{-12} s
(c) 1.6×10^{-10} s (d) 2.9×10^{-9} s

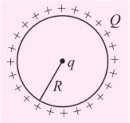
2. A conducting sphere of radius 10 cm has an unknown charge. If the electric field at a distance 20 cm from the centre of the sphere is 1.2×10^3 N C^{-1} and points radially inwards. The net charge on the sphere is

(a) -4.5×10^{-9} C (b) 4.5×10^9 C
(c) -5.3×10^{-9} C (d) 5.3×10^9 C

3. An oil drop of 10 excess electrons is held stationary under a constant electric field of 3.65×10^4 N C^{-1} in Millikan's oil drop experiment. The density of oil is 1.26 g cm^{-3}. Radius of the oil drop is (Take, $g = 9.8$ m s^{-2}, $e = 1.6 \times 10^{-19}$ C)

(a) 1.0×10^{-6} m (b) 4.8×10^{-5} m
(c) 4.8×10^{-18} m (d) 1.13×10^{-18} m

4. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring as shown in figure.



Then

(a) if $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.
(b) if $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.
(c) if $q < 0$ it will perform SHM for small displacement along the axis.
(d) all of the above.

5. Under the action of a given coulombic force the acceleration of an electron is 2.5×10^{22} m s^{-2}. Then the magnitude of the acceleration of a proton under the action of same force is nearly

(a) 1.6×10^{-19} m s^{-2} (b) 9.1×10^{31} m s^{-2}
(c) 1.5×10^{19} m s^{-2} (d) 1.6×10^{27} m s^{-2}

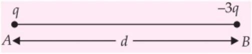
6. A rod of length 2.4 m and radius 4.6 mm carries a negative charge of 4.2×10^{-7} C spread uniformly over its surface. The electric field near the mid-point of the rod, on its surface is

(a) -8.6×10^5 N C^{-1} (b) 8.6×10^4 N C^{-1}
(c) -6.7×10^5 N C^{-1} (d) 6.7×10^4 N C^{-1}

7. The ratio of magnitude of electrostatic force and gravitational force between an electron and a proton is

(a) 6.6×10^{39} (b) 2.3×10^{39}
(c) 6.6×10^{29} (d) 2.3×10^{29}

8. Two charges q and $-3q$ are placed fixed on x-axis separated by distance d. Where should a third charge $2q$ be placed such that it will not experience any force?



(a) $\frac{d-\sqrt{3}d}{2}$ (b) $\frac{d+\sqrt{3}d}{2}$
(c) $\frac{d+3d}{2}$ (d) $\frac{d-3d}{2}$

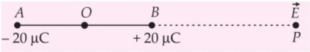
9. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is
(a) $-24\pi a\varepsilon_0 r$ (b) $-6a\varepsilon_0 r$
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$

10. Electrical as well as gravitational effects can be thought to be caused by fields. Which of the following is true of an electrical or gravitational field ?
(a) The field concept is often used to describe contact forces.
(b) Gravitational or electric field does not always exist in the space around an object.
(c) Fields are useful for understanding forces acting through a distance.
(d) There is no way to verify the existence of a force field since it is just a concept.

11. If an object of mass 1 kg contains 4×10^{20} atoms. If one electron is removed from every atom of the solid, the charge gained by the solid in 1 g is
(a) 2.8 C (b) 6.4×10^{-2} C
(c) 3.6×10^{-3} C (d) 9.2×10^{-4} C

12. Two point charges of 1 μC and –1 μC are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the midpoint and on the perpendicular bisector of the line joining the two charges. The electric field at P will be
(a) 9 N C^{-1} (b) 0.9 N C^{-1}
(c) 90 N C^{-1} (d) 0.09 N C^{-1}

13. Two charges ± 20 μC are placed 10 mm apart. The electric field at point P, on the axis of the dipole 10 cm away from its centre O on the side of the positive charge is

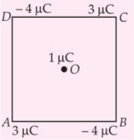


(a) 8.6×10^9 N C^{-1} (b) 4.1×10^6 N C^{-1}
(c) 3.6×10^6 N C^{-1} (d) 4.6×10^5 N C^{-1}

14. In a field free region, two electrons are released to move on a line towards each other with velocities 10^6 m s^{-1}. The distance of their closest approach will be nearer to
(a) 1.28×10^{-10} m (b) 1.92×10^{-10} m
(c) 2.56×10^{-10} m (d) 3.84×10^{-10} m

15. A uniform electric field $E = 2 \times 10^3$ N C^{-1} is acting along the positive x-axis. The flux of this field through a square of 10 cm side whose plane is parallel to the yz plane is
(a) 20 N C^{-1} m^2 (b) 30 N C^{-1} m^2
(c) 10 N C^{-1} m^2 (d) 40 N C^{-1} m^2

16. Four point charges are placed at the corners of a square $ABCD$ of side 10 cm, as shown in figure. The force on a charge of 1 μC placed at the centre of square is



(a) 7 N
(b) 8 N
(c) 2 N
(d) zero

17. The electric field at a point is
(a) always continuous
(b) continuous if there is no charge at that point
(c) discontinuous if there is a charge at that point
(d) both (b) and (c) are correct

18. If there were only one type of charge in the universe, then
(a) $\oint_s \vec{E}.\vec{ds} \neq 0$ on any surface
(b) $\oint_s \vec{E}.\vec{ds} = 0$ if the charge is outside the surface
(c) $\oint_s \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface
(d) both (b) and (c) are correct

19. A uniformly charged conducting sphere of 4.4 m diameter has a surface charge density of 60 μC m^{-2}. The charge on the sphere is
(a) 7.3×10^{-3} C (b) 3.7×10^{-6} C
(c) 7.3×10^{-6} C (d) 3.7×10^{-3} C

20. The nucleus of helium atom contains two protons that are separated by distance 3.0×10^{-15} m. The magnitude of the electrostatic force that each proton exerts on the other is
(a) 20.6 N (b) 25.6 N
(c) 15.6 N (d) 12.6 N

21. A dipole of electric dipole moment p is placed in a uniform electric field of strength E. If θ is the angle between positive directions of p and E, then the potential energy of the electric dipole is largest when θ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) zero

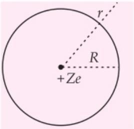
22. If a wire is stretched to make it 0.1% longer, its resistance will
(a) increase by 0.05% (b) increase by 0.2%
(c) decrease by 0.2% (d) decrease by 0.05%

HIGHER ORDER THINKING SKILLS QUESTIONS (HOTS)

23. A coin is made up of Al and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges. The magnitude of these charges is (atomic mass of Al = 26.98 g)

(a) 3.47×10^4 C (b) 3.47×10^2 C
(c) 1.67×10^{20} C (d) 1.67×10^{22} C

24. An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge upto a radius R. The atom as a whole is neutral. The electric field at a distance r from the nucleus is ($r > R$)



(a) $\frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]$

(b) $\frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{r^3} - \frac{r}{R^2}\right]$

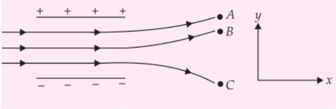
(c) $\frac{Ze}{4\pi\varepsilon_0}\left[\frac{r}{R^3} - \frac{1}{r^2}\right]$

(d) $\frac{Ze}{4\pi\varepsilon_0}\left[\frac{r}{R^3} + \frac{1}{r^2}\right]$

25. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is however not constant, but increases uniformly along the positive z-direction at the rate of 10^5 N C^{-1} m^{-1}. The force experienced by the system having a total dipole moment equal to 10^{-7} C m in the negative z– direction is

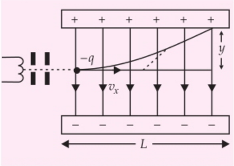
(a) -10^{-2} N (b) 10^{-2} N
(c) 10^{-4} N (d) -10^{-4} N

26. The tracks of three charged particles in a uniform electrostatic field is shown in the figure. Which particle has the highest charge to mass ratio?



(a) A (b) B (c) C (d) A and B

27. A particle of mass m and charge $-q$ enters the region between the two charged plates initially moving along x-axis with speed v_x as shown in figure. The length of plate is L and a uniform electric field E is maintained between the plates. The vertical deflection of the particle at the far edge of the plate is



(a) $\frac{qEL^2}{2mv_x^2}$ (b) $\frac{qEL^2}{2mv_x}$ (c) $\frac{2mv_x^2}{qEL^2}$ (d) $\frac{2mv_x}{qE^2L}$

28. Take the particle in question number 27 an electron projected with velocity $v_x = 4 \times 10^6$ m s^{-1}. If electric field between the plates separated by 1 cm is 8.2×10^2 N C^{-1}, then the electron will strike the upper plate at (Take $m_e = 9.1 \times 10^{-31}$ kg)

(a) 2.14 cm (b) 3.9 cm
(c) 1.23 cm (d) 3.3 cm

29. Three charges of equal magnitude q is placed at the vertices of an equilateral triangle of side l. The force on a charge Q placed at the centroid of the triangle is

(a) $\frac{3Qq}{4\pi\varepsilon_0 l^2}$ (b) $\frac{2Qq}{4\pi\varepsilon_0 l^2}$

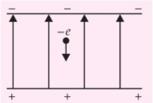
(c) $\frac{Qq}{2\pi\varepsilon_0 l^2}$ (d) zero

30. A cup contains 250 g of water. Find the number of positive charges present in the cup of water.

(a) 1.34×10^{19} C
(b) 1.34×10^7 C
(c) 2.43×10^{19} C
(d) 2.43×10^7 C

SOLUTIONS

1. **(d):** In figure the field is upward. So the negatively charged electron experiences a downward force.



$\therefore$ The acceleration of electron is $a_e = \dfrac{eE}{m_e}$ ...(i)

The time required by the electron to fall through a distance h is

$$t_e = \sqrt{\left(\frac{2h}{a_e}\right)} = \sqrt{\frac{2hm_e}{eE}} \quad \text{(using (i))}$$

$$= \left[\frac{2\times1.5\times10^{-2}\times9.11\times10^{-31}}{1.6\times10^{-19}\times2\times10^4}\right]^{1/2} = 2.9\times10^{-9}\text{ s}$$

2. **(c) :** Here, radius of sphere = 10 cm

Distance of point from the centre of the sphere, r = 20 cm = 0.2 m

Electric field, $E = 1.2 \times 10^3$ N C^{-1}

As $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$\therefore\ q = (4\pi\varepsilon_0 r^2)\ E$

$$= \frac{(0.2)^2\times(-1.2\times10^3)}{9\times10^9} = -5.3\times10^{-9}\text{ C}$$

3. **(a) :** Here, $n = 10$, $E = 3.65 \times 10^4$ N C^{-1}

$\rho_{oil} = 1.26$ g cm^{-3} = 1.26×10^3 kg m^{-3}

As the droplet is stationary,

weight of droplet = force due to electric field

or $\dfrac{4}{3}\pi r^3 \rho g = Ene$

$\therefore\ r^3 = \dfrac{3Ene}{4\pi\rho g}$

$$\therefore\ r^3 = \frac{3\times3.65\times10^4\times10\times1.6\times10^{-19}}{4\times3.14\times1.26\times10^3\times9.8}$$

$= 1.13 \times 10^{-18}$

or, $r = (1.13 \times 10^{-18})^{1/3} = 1.0 \times 10^{-6}$ m

4. **(d):** At the centre of the ring, $E = 0$ when a positive charge ($q > 0$) is displaced away from the centre in the plane of the ring, say to the right, force of repulsion on q, due to charge on right half increases and due to charge on left half decreases. Therefore, charge q is pushed back towards the centre. So option (a) is correct.

When charge q is negative ($q < 0$), force is of attraction. Therefore, charge q displaced to the right continues moving to the right till it hits the ring.

Along the axis of the ring, at a distance r from the centre,

$$E = \frac{Qr}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

If charge q is negative ($q < 0$), it will perform SHM for small displacement along the axis.

5. **(c) :** The acceleration of the electron due to given coulombic force F is

$$a_e = \frac{F}{m_e} \quad \text{...(i)}$$

where m_e is the mass of the electron.

The acceleration of the proton due to same force F is

$$a_p = \frac{F}{m_p} \quad \text{...(ii)}$$

where m_p is the mass of the proton.

Divide (ii) by (i), we get

$$\frac{a_p}{a_e} = \frac{m_e}{m_p}$$

$$a_p = \frac{a_e m_e}{m_p} = \frac{(2.5\times10^{22}\text{ m s}^{-2})(9.1\times10^{-31}\text{ kg})}{(1.67\times10^{-27}\text{ kg})}$$

$= 13.6 \times 10^{18}$ m s^{-2} $\approx 1.5 \times 10^{19}$ m s^{-2}

6. **(c) :** Here, $l = 2.4$ m, $r = 4.6$ mm $= 4.6 \times 10^{-3}$ m

$q = -4.2 \times 10^{-7}$ C

Linear charge density, $\lambda = \dfrac{q}{l}$

$$= \frac{-4.2\times10^{-7}}{2.4} = -1.75\times10^{-7}\text{ C m}^{-1}$$

Electric field, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$= \frac{-1.75\times10^{-7}}{2\times3.14\times8.854\times10^{-12}\times4.6\times10^{-3}}$$

$= -6.7 \times 10^5$ N C^{-1}

7. **(b):** Here, between an electron and a proton

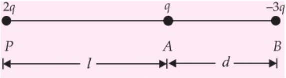
Electrostatic force, $|F_e| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r^2}$

Gravitational force, $|F_g| = \dfrac{Gm_e m_p}{r^2}$

$$\therefore\ \frac{|F_e|}{|F_g|} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{Gm_e m_p}$$

$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9\times10^{-31}\times1.66\times10^{-27}} = 2.3\times10^{39}$$

8. **(b) :**



Let a charge $2q$ be placed at P, at a distance l from A where charge q is placed, as shown in figure. The charge $2q$ will not experience any force, when force of repulsion on it due to q is balanced by force of attraction on it due to $-3q$ at B where $AB = d$

i.e., $\dfrac{(2q)(q)}{4\pi\varepsilon_0 l^2} = \dfrac{(2q)(3q)}{4\pi\varepsilon_0 (l+d)^2}$

$(l + d)^2 = 3l^2$

or $2l^2 - 2ld - d^2 = 0$

$\therefore \quad l = \dfrac{2d \pm \sqrt{4d^2 + 8d^2}}{4} = \dfrac{d}{2} \pm \dfrac{\sqrt{3}d}{2}$

$l = \dfrac{d + \sqrt{3}d}{2}$

9. (d) : $\phi = ar^2 + b$

Electric field, $E = \dfrac{-d\phi}{dr} = -2ar$ ...(i)

According to Gauss's theorem,

$\oint \vec{E} \cdot d\vec{S} = \dfrac{q_{\text{inside}}}{\varepsilon_0}$

or $-2ar 4\pi r^2 = \dfrac{q_{\text{inside}}}{\varepsilon_0}$ (Using (i))

$q_{\text{inside}} = -8\varepsilon_0 a\pi r^3$

Charge density inside the ball is

$\rho_{\text{inside}} = \dfrac{q_{\text{inside}}}{\frac{4}{3}\pi r^3} \quad \therefore \quad \rho_{\text{inside}} = \dfrac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$

$\rho_{\text{inside}} = -6a\varepsilon_0$

10. (c)

11. (b): Here, number of electrons removed = number of atoms in 1 g

or $n = \dfrac{4 \times 10^{20}}{10^3} = 4 \times 10^{17}$

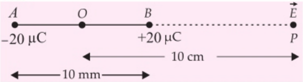
$\therefore$ charge, $q = ne = 4 \times 10^{17} \times 1.6 \times 10^{-19}$ C $= 6.4 \times 10^{-2}$ C

12. (d): The point lies on equatorial line of a short dipole.

$\therefore \quad E = \dfrac{ql}{4\pi\varepsilon_0 r^3} = \dfrac{9 \times 10^9 \times 10^{-6} \times 10^{-8}}{(10^{-1})^3}$

$= 9 \times 10^{-2}$ N C^{-1}

13. (c) :



Here, $q = \pm 20\ \mu C = \pm 20 \times 10^{-6}$ C

or $2a = 10$ mm $= 10 \times 10^{-3}$ m

$r = OP = 10$ cm $= 10 \times 10^{-2}$ m

$|\vec{p}| = q \times 2a = 20 \times 10^{-6} \times 10 \times 10^{-3}$ m $= 2 \times 10^{-7}$ Cm

The electric field along BP, $E = \dfrac{2\vec{p}\,r}{4\pi\varepsilon_0 (r^2 - a^2)^2}$

As $a \ll r$,

$E = \dfrac{2|\vec{p}|}{4\pi\varepsilon_0 r^3} = \dfrac{2 \times 2 \times 10^{-7} \times 9 \times 10^9}{(10 \times 10^{-2})^3}$

$= 3.6 \times 10^6$ N C^{-1}

14. (c) : At the distance of closest approach (r)

K.E. = P.E.

$\therefore \quad \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$

$mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$

$r = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{mv^2} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{9 \times 10^{-31} \times (10^6)^2}$

$= 2.56 \times 10^{-10}$ m

15. (a) : Here, $E = 2 \times 10^3$ N C^{-1} is along + x-axis

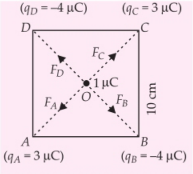
Surface area, $S = (10\text{ cm})^2 = 10^2 \times 10^{-4}\text{ m}^2 = 10^{-2}\text{ m}^2$.

When plane is parallel to yz plane, $\theta = 0°$

So $\phi = E\,S\cos\theta = 2 \times 10^3 \times 10^{-2}\cos 0°$

$= 20$ N C^{-1} m^2

16. (d):



From figure, length of diagonal of the square

$= AC = BD = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ cm

$OA = OB = OC = OD = \dfrac{10\sqrt{2}}{2} = \dfrac{10}{\sqrt{2}}$ cm

Forces of repulsion on 1 μC charge at O due to 3 μC charges, at A and C are equal and opposite. So they cancel each other.

Similarly, forces of attraction on 1 μC charge at O due to –4 μC charges at B and D are also equal and opposite. So they also cancel each other.

Hence the net force on the charge of 1 μC at O is zero.

17. (d): Electric field at a point is continuous if there is no charge at that point. And the field is discontinuous if there is charge at that point. So both options (b) and (c) are correct.

18. (d): According to Gauss's theorem in electrostatics

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

Here q is charge enclosed by the surface.

If the charge is outside the surface, then $q_{inside} = 0$

Also, $\oint \vec{E}.d\vec{s} = 0$.

So, both (b) and (c) are correct.

19. (d): Here, $D = 2r = 4.4$ m, *or* $r = 2.2$ m

$\sigma = 60\ \mu C\ m^{-2}$

Charge on the sphere, $q = \sigma \times 4\pi r^2$

$$= 60\times10^{-6}\times4\times\frac{22}{7}\times(2.2)^2 = 3.7\times10^{-3}\ C$$

20. (b): Charge of proton, $q_p = 1.6 \times 10^{-19}$ C

Distance between the protons,

$r = 3 \times 10^{-15}$ m

The magnitude of electrostatic force between protons is

$$F_e = \frac{q_p\, q_p}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{(3\times10^{-15})^2}$$

$= 25.6$ N

21. (c) : The potential energy of an electric dipole in a uniform electric field is

$U = -\vec{p}\cdot\vec{E}$

$U = -pE\cos\theta$

For U to be maximum

$\cos\theta = -1 \Rightarrow \theta = \pi$

22. (b) : Resistance of wire

$$R = \frac{\rho l}{A} \qquad \text{...(i)}$$

On stretching, volume (V) remains constant.

So $V = Al$ or $A = \dfrac{V}{l}$

$\therefore\ R = \dfrac{\rho l^2}{V}$ (Using (i))

Taking logarithm on both sides and differentiating we get,

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l} \qquad (\because V \text{ and } \rho \text{ are constants})$$

or $\dfrac{\Delta R}{R}\% = \dfrac{2\Delta l}{l}\%$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

23. (a) : Mass of the coin = 0.75 g, $Z_{Al} = 13$

Atomic mass of aluminium = 26.98 g

Avogadro's number = 6.02×10^{23}

Number of Al atoms in the coin,

$$N = \frac{6.02\times10^{23}}{26.98}\times0.75 = 1.67\times10^{22}$$

As charge number of Al is 13, each atom of Al contain 13 protons and 13 electrons.

Magnitude of positive and negative charges is one coin

$= NZ_{Al}\,e$

$= 1.67 \times 10^{22} \times 13 \times 1.6 \times 10^{-19}$ C

$= 3.47 \times 10^4$ C

24. (a) : Charge on nucleus = + Ze

Total negative charge = – Ze

($\because$ atom is electrical neutral]

Negative charge density, $\rho = \dfrac{\text{charge}}{\text{volume}}$

$$= \frac{-Ze}{\frac{4}{3}\pi R^3}$$

i.e. $\rho = -\dfrac{3}{4}\dfrac{Ze}{\pi R^3}$ ...(i)

Consider a Gaussian surface with radius r.

By Gauss's theorem

$$\phi = E(r)\times4\pi r^2 = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

Charge enclosed by Gaussian surface

$$q = Ze + \frac{4\pi r^3}{3}\rho = Ze - Ze\frac{r^3}{R^3} \qquad \text{(Using (i)}$$

From (ii)

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{Ze - Ze\frac{r^3}{R^3}}{4\pi\varepsilon_0 r^2} = \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]$$

25. (a) : Consider an electric dipole with $-q$ charge at A and $+q$ charge at B, placed along z-axis, such that its dipole moment is in negative z direction.

i.e., $p_z = -10^{-7}$ Cm

The electric field is along positive direction of z-axis, such that

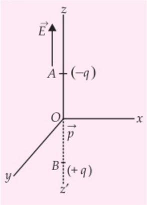
$$\frac{dE}{dz} = 10^5\ N\ C^{-1}\ m^{-1}$$

From $F = qdE$

$$= (q\times dz)\times\frac{dE}{dz}$$

$$= p\frac{dE}{dz}$$

Force experienced by the system in the negative z-direction



$$F = -p \times \left(-\frac{dE}{dz}\right)$$

$= 10^{-7} \times (-10^{5}) = -10^{-2}$ N

26. (c) : Particles A and B have negative charges because they are being deflected towards the positive plate of the electrostatic field. Particle C has positive charge because it is being deflected towards the negative plate.

$\therefore$ Deflection of charged particle in time t in y-direction

$$h = 0 \times t + \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2$$

i.e. $h \propto q/m$

As the particle C suffers maximum deflection in y-direction, so it has highest charge to mass q/m ratio.

27. (a) : Here, in the vertical direction, initial velocity, $v = 0$

acceleration, $a = \frac{F}{m} = \frac{qE}{m}$ ...(i) ($\because$ $F = qE$)

Time taken to cross the field,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{L}{v_x} \quad \text{...(ii)}$$

($\because$ velocity along the horizontal direction is constant)

Now, $s = vt + \frac{1}{2}at^2$

$\therefore$ deflection, $y = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$

[Using (i) and (ii)]

$$\therefore \quad y = \frac{qEL^2}{2mv_x^2} \quad \text{...(iii)}$$

28. (d) : Given: $v_x = 4 \times 10^6$ m s^{-1},

$d = 1$ cm $= 1 \times 10^{-2}$ m, $E = 8.2 \times 10^2$ N C^{-1}

$q = e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.

The electron will strike the upper plate at its other end of $x = L$ as soon as its deflection.

or $y = \frac{d}{2} = \frac{10^{-2}}{2}\text{ m} = 5 \times 10^{-3}\text{ m}$

From equation (iii),

$$L = \sqrt{\frac{2m_e v_x^2 y}{qE}}$$

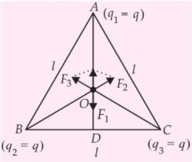
$$= \sqrt{\frac{2 \times 9.1 \times 10^{-31} \times (4 \times 10^6)^2 \times 5 \times 10^{-3}}{1.6 \times 10^{-19} \times 8.2 \times 10^2}}$$

$= 3.33 \times 10^{-2}$ m

$L = 3.33$ cm

The electrons will strike the upper plate at its other end at 3.33 cm

29. (d) : As shown in figure draw, $AD \perp BC$.



$\therefore$ $AD = AB\cos 30° = \frac{l\sqrt{3}}{2}$

Distance AO of the centroid O from A

$= \frac{2}{3}\text{AD} = \frac{2l}{3}\frac{\sqrt{3}}{2} = \frac{l}{\sqrt{3}}$

$\therefore$ Force on Q at O due to charge $q_1 = q$ at A

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(l/\sqrt{3})^2} = \frac{3Qq}{4\pi\varepsilon_0 l^2}\text{, along } AO$$

Similarly, force on Q due to charge $q_2 = q$ at B

$$F_2 = \frac{3Qq}{4\pi\varepsilon_0 l^2} \text{ along } BO$$

and force on Q due to charge $q_3 = q$ at C

$$F_3 = \frac{3Qq}{4\pi\varepsilon_0 l^2}\text{, along } CO$$

Angle between forces F_2 and $F_3 = 120°$

By parallelogram law, resultant of F_2 and F_3

$= \frac{3Qq}{4\pi\varepsilon_0 l^2}$ along OA

$\therefore$ Total force on $Q = \frac{3Qq}{4\pi\varepsilon_0 l^2} - \frac{3Qq}{4\pi\varepsilon_0 l^2} = 0$

30. (b) : Mass of water = 250 g

Molecular mass of water = 18 g

Number of molecules in 18 g of water (Avogadro's Number) $= 6.02 \times 10^{23}$

$\therefore$ Number of molecules in one cup of water

$$= \frac{250}{18} \times 6.02 \times 10^{23}$$

Each molecule of water contains two hydrogen atoms and one oxygen atom, i.e. 10 electrons and 10 protons.

$\therefore$ Total positive and negative charge has the same magnitude and is

$$= \frac{250}{18} \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19}\text{ C}$$

$= 1.34 \times 10^7$ C

BRAIN MAP

RAY OPTICS and OPTICAL INSTRUMENTS

Ray Optics

Image Formation by Spherical Mirrors

- For any spherical mirror,
$$f = \frac{R}{2}$$
- Mirror formula,
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$$
- Magnification m is –ve for real images and +ve for virtual images.
- f and R –ve for a concave mirror and +ve for a convex mirror.
- For a real object u is –ve, v is –ve for real image and +ve for virtual image.

Refraction of light, Lateral shift and Real and Apparent Depths

- Refractive index,
$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}}$$
$$\mu = \frac{c}{v}$$
- When light travels from medium a to medium b,
$$^{a}\mu_b = \frac{\mu_b}{\mu_a} = \frac{\sin i}{\sin r}$$
- $^{a}\mu_b \times {}^{b}\mu_c = {}^{a}\mu_c$
- $\mu = \dfrac{\text{real depth}(x)}{\text{apparent depth }(y)}$
- Lateral shift, $d = t\dfrac{\sin(i-r)}{\cos r}$
- Displacement of image $= x - y$

Total Internal Reflection

- Critical angle, i_c = Angle of incidence in denser medium for which angle of refraction is 90° in rarer medium.
- Refractive index of denser medium,
$$\mu = \frac{1}{\sin i_c}$$
- Total internal reflection occurs when $i > i_c$

Refraction from Spherical Surfaces

- When refraction occurs from rarer to denser medium,
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$
- When refraction occurs from denser to rarer medium,
$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$
- First principal focal length,
$$f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$$
- Second principal focal length,
$$f_2 = -\frac{\mu_2 R}{\mu_2 - \mu_1}$$

Lens Maker's Formula

- For the lens of material of refractive index μ_2 placed in a medium of refractive index μ_1,
$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
- When the lens is placed in air
$\mu_1 = 1$ and $\mu_2 = \mu$
$$\therefore \quad \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Combination of Lenses

- Focal length F of a combination of two lenses of focal length f_1 and f_2 separated by a distance d is
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$
- If the lenses are in contact, $d = 0$
$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power of Lenses

- Power of a lens,
$$P = \frac{1}{f\text{ (in m)}} = \frac{100}{f\text{ (in cm)}}$$
- Magnification of combination of lenses,
$m = m_1 \times m_2 \times m_3 .....$
- Power of two lenses separated by a distance d is
$$P = P_1 + P_2 - dP_1P_2$$

Refraction and Dispersion of Light through a Prism

- For refraction, through a prism
$A + \delta = i + i'$ and $r + r' = A$
- In the condition of minimum deviation,
$i = i'$, $r = r'$ and $\delta = \delta_m$
$$\mu = \frac{\sin\dfrac{A + \delta_m}{2}}{\sin\dfrac{A}{2}}$$
- Deviation produced by a prism of small angle $\delta = (\mu - 1)A$
- Angular dispersion $= \delta_V - \delta_R = (\mu_V - \mu_R)A$
- Dispersive power,
$$\omega = \frac{\delta_V - \delta_R}{\delta} = \frac{\mu_V - \mu_R}{\mu - 1}$$
- Mean deviation,
$$\delta = \frac{\delta_V + \delta_R}{2}$$

Optical Instruments

The Defects of Vision

- Myopia or short sightedness : For observing distant objects, a concave lens of focal length $f = -x$ is to be used, where x is distance of far point of defective eye.
- Hypermetropia or far sightedness : For observing near object, a convex lens of focal length f is to be used where
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
Here, $u = d$ = least distance of distinct vision of normal eye and $v = -x$ = distance of near point of defective eye

Simple Microscope

- When the final image is formed at the least distance of distinct vision, the magnifying power
$$M = 1 + \frac{D}{f}$$
- When the final image is formed at infinity, the magnifying power
$$M = \frac{D}{f}$$

Compound Microscope

- Magnifying power, $M = m_o \times m_e$
- When the final image is formed at the least distance of distinct vision
$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$
- When the final image is formed at infinity
$$M = \frac{v_o}{u_o} \cdot \frac{D}{f_e} = -\frac{L}{f_o} \cdot \frac{D}{f_e}$$

Telescope

- Astronomical telescope: (i) In normal adjustment,
$$M = \frac{f_o}{f_e}$$
Distance between objective and eyepiece $= f_o + f_e$
(ii) When the final image is formed at the least distance of distinct vision,
$$M = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$
Distance between objective and eyepiece
$$= f_o + u_e = f_o + \frac{f_e D}{f_e + D}$$
- Terrestrial telescope. (i) In normal adjustment:
$$M = \frac{f_o}{f_e}$$
Distance between objective and eyepiece $= f_o + 4f + f_e$
where f = focal length of the erecting lens.
- Galileo's telescope. In normal adjustment,
$$M = \frac{f_o}{f_e}$$
Distance between objective and eyepiece $= f_o - f_e$
- Reflecting telescope:
$$M = \frac{f_o}{f_e} = \frac{R/2}{f_e}$$

Foundation Series

JEE

Maximize your chance of success, and high rank in JEE (Main and Advanced) /BITSAT by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

UNIT-6 Electrostatics | Current Electricity

ELECTRIC CHARGE

It is an intrinsic property of the elementary particles of matter, which all the objects are made up of. It is because of these electric charges that various objects exert strong electric forces of attraction or repulsion on each other. Electric charge is invariant (It does not depend on speed.)

Electric charge is a scalar quantity. Its SI unit is coulomb (C).

In CGS electrostatic system, the unit of charge is electrostatic unit (esu) or stat coulomb (stat C).

$$1 \text{ C} = 3 \times 10^9 \text{ stat C}$$

In CGS electromagnetic system, the unit of charge is electromagnetic unit (emu).

$$1 \text{ C} = \frac{1}{10} \text{ emu}$$

A proton has a positive charge ($+e$) and an electron has a negative charge ($-e$), where $e = 1.6 \times 10^{-19}$ C

Basic Properties of Electric Charge

Charge is transferable : If a charged body is put in contact with an uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

Charge is always associated with mass, *i.e.*, charge can not exist without mass though mass can exist without charge. So, the presence of charge itself is a convincing proof of existence of mass.

Quantisation of charge : Total charge on a body is always an integral multiple of a basic unit of charge denoted by e and is given by $q = ne$

where n is any integer, positive or negative.

The quantisation of charge was first suggested by Faraday. It was experimentally demonstrated by Millikan in 1912.

The basic unit of charge is the charge that an electron or proton carries.

Additivity of charge : Total charge of a system is the algebraic sum (*i.e.* sum is taking into account with proper signs) of all individual charges in the system.

Conservation of charge : Total charge of an isolated system remains unchanged with time. In other words, charge can neither be created nor be destroyed. Conservation of charge is found to hold good in all types of reactions either chemical or nuclear.

Charge is invariant : Charge is independent on the frame of reference.

Like charges repel each other while unlike charges attract each other.

Illustration 1 : Which is bigger – a coulomb or a charge on an electron? How many electronic charges form one coulomb of charge?

Soln.: One coulomb of charge is bigger than the charge on an electron.

Charge on one electron,

$$e = 1.6 \times 10^{-19} \text{C}.$$

$\therefore$ Number of electronic charges in 1 coulomb,

$$n = \frac{q}{e} = \frac{1 \text{ C}}{1.6 \times 10^{-19} \text{C}} = 6.25 \times 10^{18}$$

COULOMB'S LAW

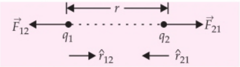
The force of attraction or repulsion between two stationary point charges is directly proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance between them. This force acts along the line joining the two charges.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{k q_1 q_2}{r^2}$$

Here, $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$

where, ε_0 is permittivity of free space.

In vector form, coulomb's law



$\vec{F}_{21}$ = Force on charge q_2 due to q_1

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where, $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{r}$ is a unit vector in the direction from q_1 to q_2.

Similarly, $\vec{F}_{12}$ = Force on charge q_1 due to q_2

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21},$$

where $\hat{r}_{21} = \dfrac{\vec{r}_{21}}{r}$ is a unit vector in the direction from q_2 to q_1.

Dielectric Constant or Relative Permittivity

The relative permittivity or dielectric constant of a medium may be defined as the ratio of the force between two charges placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium.

$$F_{\text{medium}} = \frac{F_{\text{vacuum}}}{K} \quad \text{or} \quad K = \frac{F_{\text{vacuum}}}{F_{\text{medium}}} = \frac{\varepsilon}{\varepsilon_0}$$

K for vacuum = 1

K for air = 1.00054

K for water = 80

Comparison between Coulomb Force and Gravitational Force

Coulomb force and gravitational force follow the same inverse square law.

Coulomb force can be attractive or repulsive while gravitational force is always attractive.

Coulomb force between the two charges depends on the medium between two charges while gravitational force is independent of the medium between the two bodies.

- The ratio of coulomb force to the gravitational force between two protons at a distance r apart is

$$\frac{e^2}{4\pi\varepsilon_0 G m_p m_p} \approx 1.3 \times 10^{36}$$

The ratio of coulomb force to the gravitational force between proton and electron at a distance r apart is

$$\frac{e^2}{4\pi\varepsilon_0 G m_e m_p} \approx 2.3 \times 10^{39}$$

Principle of Superposition

According to principle of superposition, force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. In a system of charges q_1, q_2 ...... q_n, the total force $\vec{F}_1$ on the charge q_1, due to all other charges, is then given by the vector sum of the forces $\vec{F}_{12}, \vec{F}_{13}, \ldots\ldots\ldots\ldots, \vec{F}_{1n}$.

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \ldots + \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{i1}^2}\hat{r}_{i1}$$

Continuous Charge Distribution

Linear charge density : Charge per unit length is called linear charge density. It is denoted by symbol λ.

$$\lambda = \frac{\text{Charge}}{\text{Length}}$$

Its SI unit is C m^{-1}.

Surface charge density : Charge per unit area is called surface charge density. It is denoted by symbol σ.

$$\sigma = \frac{\text{Charge}}{\text{Area}}$$

Its SI unit is C m^{-2}.

Volume charge density : Charge per unit volume is called volume charge density. It is denoted by symbol ρ.

$$\rho = \frac{\text{Charge}}{\text{Volume}}$$

Its SI unit is C m^{-3}.

Illustration 2 : Two equally charged identical metal spheres A and B repel each other with a force of 2.0×10^{-5} N. A third identical uncharged sphere C is touched to A, then placed at the mid-point between A and B. Calculate the net electrostatic force on C.

Soln.: Let the charge on each of the spheres A and B be q. If the separation between A and B is r, then electrostatic force between spheres A and B will be

$$F = \frac{kq^2}{r^2} = 2.0 \times 10^{-5} \text{N}.$$

When sphere C is touched to A, the spheres share charge $q/2$ each, because both are identical.

$\therefore$ Force on C due to A

$$= k\frac{(q/2)^2}{(r/2)^2} = k\frac{q^2}{r^2}, \text{ along } AC.$$

Force on C due to B

$$\frac{kq \cdot q/2}{(r/2)^2} = \frac{k2q^2}{r^2}, \text{ along } BC$$

Since these forces are in opposite directions therefore, net force on C is

$$F' = \frac{k \cdot 2q^2}{r^2} - \frac{k \cdot q^2}{r^2}$$

ELECTRIC FIELD

An electric field is said to exist at a point if a force of electric origin is exerted on a stationary charged body placed at that point.

Quantitatively, the electric field or the electric intensity or the electric field strength, $\vec{E}$ at a point is defined as the force experienced by a unit positive test charge placed at that point, without disturbing the position of source charge.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Here, q_0 is test charge.

The direction of $\vec{E}$ is the same in the direction of $\vec{F}$.

Units and dimensions of electric field

As the electric field is force per unit charge, so its SI unit is Newton per coulomb (N C^{-1}). It is equivalent to volt per metre (V m^{-1}).

The dimension of $\vec{E}$ is [$MLT^{-3}A^{-1}$].

Illustration 3 : Calculate the electric field strength required to just support a water drop of mass 10^{-3} kg and having a charge 1.6×10^{-19} C.

Soln.: Here, $m = 10^{-3}$ kg, $q = 1.6 \times 10^{-19}$ C

Let E be the strength of the electric field required to just support the water drop. Then

Force on water drop due to electric field = weight of water drop *i.e.*, $qE = mg$

$$\therefore \quad E = \frac{mg}{q} = \frac{10^{-3} \times 9.8}{1.6 \times 10^{-19}} = 6.125 \times 10^{16} \text{ N C}^{-1}$$

ELECTRIC FIELD LINES OR ELECTRIC LINE OF FORCE

An electric line of force may be defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of the electric field at that point

Properties of Electric Lines of Force :

- The lines of force are continuous smooth curves without any breaks.
- The lines of force start at positive charges and end at negative charges. They cannot form closed loops. If there is a single charge, then the lines of force will start or end at infinity.
- The tangent to a line of force at any point gives the direction of the electric field at that point.
- No two lines of force cross each other.
- The lines of force are always normal to the surface of a conductor on which the charges are in equilibrium.
- The lines of force have a tendency to expand laterally so as to exert a lateral pressure on neighbouring lines of force. This explains repulsion between two similar charges.
- The lines of force have a tendency to contract lengthwise. This explains attraction between two unlike charges.
- The lines of force do not pass through a conductor because the electric field inside a charged conductor is zero.

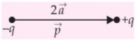
ELECTRIC DIPOLE

A pair of equal and opposite charges separated by small distance is called an electric dipole.

Dipole Moment

The dipole moment of an electric dipole is a vector whose magnitude is either charge times the separation between the two opposite charges and the direction is along the dipole axis from the negative to the positive charge.

$$\vec{p} = q \times 2\vec{a}$$



The SI unit of dipole moment is coulomb metre (C m)

Dipole Field

The electric field produced by an electric dipole is called a dipole field.

Electric field on axial line (end on position) of an electric dipole

Electric field at a point on axial line of electric dipole is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2 - a^2)^2}$$

where r is the distance of the point from the centre of the electric dipole.

For $r >> a$, $E = \dfrac{2p}{4\pi\varepsilon_0 r^3}$

The direction of the electric field on axial line of an electric dipole is along the direction of the dipole moment (*i.e.* from $-q$ to q).

Electric field on equatorial line (board on position) of an electric dipole

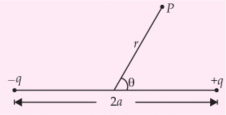
Electric field at a point on equatorial line of electric dipole is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$$

For $r >> a$, $E = \dfrac{1}{4\pi\varepsilon_0} \dfrac{p}{r^3}$

The direction of the electric field on equatorial line of the electric dipole is opposite to the direction of the dipole moment. (*i.e.* from q to $-q$).

Electric field at any point due to an electric dipole



The electric field at point P due to an electric dipole is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

Special cases :

- When the point P lies on the axial line of dipole, *i.e.*, θ, = 0°

  $\therefore \qquad E=\frac{2p}{4\pi\varepsilon_0 r^3}$

- When the point P lies on the equatorial line of dipole *i.e.* $\theta = 90°$

  $\therefore \qquad E=\frac{p}{4\pi\varepsilon_0 r^3}$

Electric Field due to a Uniformly Charged ring

Electric field at a point on the axis of uniformly charged ring at a distance r from its centre is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qr}{(r^2+a^2)^{3/2}}$$

where q is the charge on the ring and a is the radius of the ring.

For $r >> a$, $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

At the centre of the ring, $E = 0$.

Electric Dipole in a Uniform Electric Field

When an electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$, it will experience a torque and is given by

$$\vec{\tau}=\vec{p}\times\vec{E} \quad \text{or} \quad \tau = pE\sin\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.

The direction of $\vec{\tau}$ is given by right handed screw rule and is perpendicular to $\vec{p}$ and $\vec{E}$, *i.e.*, perpendicular to the plane of the paper and outwards.

Torque acting on a dipole is maximum ($\tau_{max} = pE$) when dipole is perpendicular to the field and minimum ($\tau = 0$) when dipole is parallel or antiparallel to the field.

When a dipole is placed in a uniform electric field, it will experience only torque and the net force on the dipole is zero while when it is placed in a non uniform electric field, it will experience both torque and net force.

POTENTIAL ENERGY OF AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Potential energy of an electric dipole in a uniform electric field is

$U = -pE(\cos\theta_2 - \cos\theta_1)$

where θ_1 is the initial angle between $\vec{p}$ and $\vec{E}$ and θ_2 is the final angle between $\vec{p}$ and $\vec{E}$.

If the dipole is rotated from $\theta_1 = 0°$ (aligned parallel to $\vec{E}$) to $\theta_2 = \theta$, then

$$U = -pE(\cos\theta - 1) = pE(1-\cos\theta)$$

If the dipole is rotated from $\theta_1 = 90°$ (aligned perpendicular to $\vec{E}$) to $\theta_2 = \theta$, then

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Illustration 4 : Two charges, one + 5 μC and another – 5 μC are placed 1 mm apart. Calculate the dipole moment.

Soln.: Here, $q = 5\ \mu C = 5\times10^{-6}$ C,

$2a = 1$ mm $= 10^{-3}$ m

Dipole moment,

$p = q\times 2a = 5\times10^{-6}\times10^{-3} = 5\times10^{-9}$ C m

Illustration 5 : Two point charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity at a point distant 4 cm from the midpoint on the axial line of the dipole.

Soln.: Here, $q = 5\times10^{-6}$ C, $2a = 0.04$ m,

$a = 0.02$ m, $r = 0.04$ m

$$E_{\text{axial}}=\frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{2(q\times 2a)r}{(r^2-a^2)^2}$$

$$=\frac{9\times10^9\times2\times5\times10^{-6}\times0.04\times0.04}{[(0.04)^2-(0.02)^2]^2}$$

or Electric field intensity $=\frac{144}{144\times10^{-8}}=10^8\ \text{N C}^{-1}$.

Illustration 6 : Two charges ± 1000 μC are separated by 2mm. The dipole formed is held at an angle of 30° with a uniform electric field of 15 × 10⁴ NC⁻¹. Calculate the torque acting on the dipole.

Soln.: Here, $q = \pm 1000\ \mu C = 10^{-3}$ C, $2a = 2$mm

$= 2\times10^{-3}$ m, $\theta = 30°$

$\therefore \tau = pE\sin\theta = q(2a)\times E\sin\theta$

$= 10^{-3}\times2\times10^{-3}\times15\times10^4\sin30° = 0.15$ Nm

ELECTRIC FLUX

Electric flux $\Delta\phi$ through an area element $\Delta\vec{S}$ in an electric field $\vec{E}$ is defined as

$$\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\theta$$

where θ is the angle between $\vec{E}$ and outward normal to the area element.

It represent the number of electric field lines passing the area element.

Electric flux is a scalar quantity.

Gauss's law

It states that the total electric flux linked with a closed surface is $1/\varepsilon_0$ times the total charge enclosed by the closed surface.

$$\oint_s \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

where q is total charge enclosed by the closed surface S.

The term q on the right side of Gauss's law includes the sum of all charges enclosed by the closed surface. The charges may be located anywhere inside the closed surface.

The total electric flux through a closed surface is zero if no charge is enclosed by the surface.

Gauss's law is true for any closed surface, regardless of its shape or size.

The surface that we choose for the application of Gauss's law is called the **Gaussian surface.**

In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field (whose flux is calculated) is due to all the charges, both inside and outside the closed surface. However, the term (q) represents only the total charge inside the closed surface.

Gauss's law is based on inverse square dependence on distance.

Applications of Gauss's law

Electric field due to an infinitely long thin uniformly charged straight wire

Electric field due to thin infinitely long straight wire of uniform linear charge density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where r is the perpendicular distance of the observation point from the wire.

Electric field due to a uniformly charged thin spherical shell

Electric field due to uniformly charged thin spherical shell of uniform surface charge density σ and radius R at a point distant r from the centre of the shell is given as follows :

- At a point outside the shell *i.e.*, $r > R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- At a point on the surface of the shell *i.e.*, $r = R$

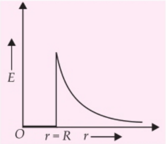
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

- At a point inside the shell *i.e.*, $r < R$

  $E = 0$

  Here, $q = 4\pi R^2\sigma$

  The variation of E with r for a uniformly charged thin spherical shell is as shown in the figure.



Electric field due to a uniformly charged non-conducting solid sphere

Electric field due to a uniformly charged non conducting solid sphere of uniform volume charge density ρ and radius R at a point distant r from the centre of the sphere is given as follows :

- At a point outside the sphere *i.e.*, $r > R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- At a point on the surface of the sphere *i.e.*, $r = R$

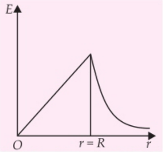
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

- At a point inside the sphere *i.e.*, $r < R$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

Here, $q = \frac{4}{3}\pi R^3\rho$

The variation of E with r for a uniform charged non conducting sphere is as shown in the figure.



Electric field due to a uniformly charged infinite thin plane sheet

Electric field due to a infinite thin plane sheet of uniformly surface charge density σ is

$$E = \frac{\sigma}{2\varepsilon_0}$$

E is in dependent of r, distance of the point from sheet.

Electric field due to two thin infinite parallel sheets of equal and opposite charges

Electric field due to two thin infinite parallel sheets of uniform surface density $+\sigma$ and $-\sigma$, is given as follows :

- At a point anywhere in the space between the two sheets

$$E = \frac{\sigma}{\varepsilon_0}$$

- At point outside the sheets, $E = 0$.

Illustration 7 : A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly over its surface. What is the electric field (i) inside the sphere, (ii) just outside the sphere, (iii) at a point 18 cm from the centre of the sphere?

Soln.: Here $q = 1.6 \times 10^{-7}$ C

R = 12 cm = 0.12 m

(i) Inside the sphere, $E = 0$. This because the charge resides on the outer surface of the spherical conductor.

(ii) Just outside the sphere, $r = R = 0.12$ m. Here the charge may be assumed to be concentrated at the centre of the sphere.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2} = 10^5 \text{ N C}^{-1}.$$

(iii) At a point 18 cm from the centre,

r = 18 cm = 0.18 m.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2}$$

$$= 4.44 \times 10^4 \text{ N C}^{-1}.$$

ELECTRIC POTENTIAL

Electric potential at a point is defined as amount of workdone in bringing a unit positive charge from infinity to that point. It is denoted by symbol V.

$$V = \frac{W}{q}$$

Electric potential is a scalar quantity.

The SI unit of potential is volt and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

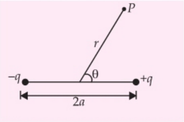
Electric potential due to a point charge q at a distance r from a charge is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

Electric potential due to system of charges : The electric potential at a point due to a system of charges is equal to the algebraic sum of the electric potentials due to individual charges at that point.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n}\right) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

Electric Potential at any Point due to an Electric Dipole



The electric potential at point P due to an electric dipole is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

Special cases :

- When the point P lies on the axial line of dipole *i.e.*, $\theta = 0°$.

  $$\therefore \quad V = \frac{p}{4\pi\varepsilon_0 r^2}$$

- When the point P lies on the equatorial line of the dipole, *i.e.*, $\theta = 90°$ $\quad \therefore \quad V = 0$.

Electric Potential due to Uniformly Charged Thin Spherical Shell

Electric potential due to a uniformly charged spherical shell of uniform surface charge density σ and radius R at a point distant r from the centre of the shell is given as follows :

- At a point outside the shell *i.e.*, $r > R$

  $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

- At a point on the surface of the shell *i.e.*, $r = R$

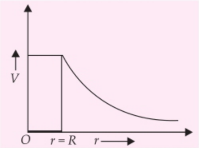
  $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

- At a point inside the shell *i.e.*, $r > R$

  $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

Here, $q = 4\pi R^2\sigma$

The variation of V with r for a uniformly charged thin spherical shell is shown in the figure.



Electric Potential due to a Uniformly Charged non-conducting Solid Sphere

Electric potential due to a uniformly charged non-conducting solid sphere of uniform volume charge density ρ and radius R at a point distant r from the centre of sphere is given as follows :

- At a point outside the sphere *i.e.*, $r > R$

  $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

- At a point on the surface of the sphere *i.e.*, $r = R$

  $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

- At a point inside the sphere *i.e.*, $r < R$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$$

Here $q = \frac{4}{3}\pi R^3\rho$

Illustration 8 : Determine the electric potential at the surface of a gold nucleus. The radius is 6.6×10^{-15} m and the atomic number $Z = 79$. Given charge on a proton $= 1.6 \times 10^{-19}$ C.

Soln.: As nucleus is spherical, it behaves like a point charge for external points.

Here, $q = ne = 79 \times 1.6 \times 10^{-19}$ C,

$r = 6.6 \times 10^{-15}$ m

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{9\times10^9\times79\times1.6\times10^{-19}}{6.6\times10^{-15}}\text{ V}$$

$$= 1.7 \times 10^7 \text{ V.}$$

Illustration 9 : A short electric dipole has dipole moment of 4×10^{-9} C m. Determine the electric potential due to the dipole at a point distance 0.3 m from the centre of the dipole situated (a) on the axial line (b) on equatorial line and (c) on a line making an angle of 60° with the dipole axis.

Soln.: Here, $p = 4 \times 10^{-9}$ C m, $r = 0.3$ m.

(a) Potential at a point on the axial line

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} = \frac{9\times10^9\times4\times10^{-9}}{(0.3)^2}$$

$$= 400 \text{ V.}$$

(b) Potential at a point on the equatorial line = 0.

(c) Potential at a point on a line that makes an angle of 60° with dipole axis

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$$

$$= \frac{9\times10^9\times4\times10^{-9}\cos60^\circ}{(0.3)^2} = 200 \text{ V.}$$

EQUIPOTENTIAL SURFACE

An equipotential surface is a surface with a constant value of potential at all points on the surface.

Properties of an Equipotential Surface

Electric field lines are always perpendicular to an equipotential surface.

Work done in moving an electric charge from one point to another on an equipotential surface is zero.

Two equipotential surfaces can never intersect one another.

Relationship between $\vec{E}$ and V

$$\vec{E} = -\vec{\nabla}V$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$

–ve sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

Electric Potential Energy

Electric potential energy of a system of charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.

The SI unit of electrical potential energy is joule.

Electric potential energy of a system of two charges is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

Electric potential energy of a system of n point charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

DIELECTRICS AND POLARISATION

Dielectrics : Dielectrics are non conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers.

Polar molecule : A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). A polar molecule has a permanent dipole moment *e.g.*, water (H_2O) and HCl.

Non-polar molecule : A non-polar molecule is one in which the centres of positive and negative charges coincide. A non polar molecule has no permanent dipole moment. *e.g.*, oxygen (O_2) and hydrogen (H_2).

Polarisation : The dipole moment per unit volume is called polarisation and is denoted by $\vec{P}$. For linear isotropic dielectrics $\vec{P} = \chi_e\vec{E}$.

where χ_e is a constant characteristic of the dielectric and is called the electric susceptibility of the dielectric medium.

CAPACITANCE

Capacitance (C) of a capacitor is the ratio of charge(Q) given and the potential (V) to which it is raised.

$$C = \frac{Q}{V}$$

The SI unit of capacitance is **farad** (F).

1 millifarad (mF) = 10^{-3} farad

1 microfarad (μF) = 10^{-6} farad

1 picofarad (pF) = 10^{-12} farad.

Capacitance is a scalar quantity.

The dimensional formula of capacitance is $[M^{-1}L^{-2}T^4A^2]$.

Capacitance of Spherical Conductor

Capacitance of a spherical conductor of radius R is

$C = 4\pi\varepsilon_0 R$

Taking earth to be a conducting sphere of radius 6400 km, its capacitance will be

$$C = 4\pi\varepsilon_0 R = \frac{6.4\times10^6}{9\times10^9} = 711\ \mu\text{F}$$

Capacitor

A condenser or a capacitor is a device that stores electric charge. It consists of two conductors separated by an insulator or dielectric. The two conductors carry equal and opposite charges $\pm Q$.

In an electrical circuit, a capacitor of fixed capacitance is represented by the symbol as shown in figure (a) while a capacitor of variable capacitance is represented by the symbol as shown in figure (b).

(a) (b)

Types of capacitors

Depending on their geometry, capacitors are classified as :

- Parallel plate capacitor
- Cylindrical capacitor
- Spherical capacitor

Parallel plate capacitor : It consists of two similar flat conducting plates, arranged parallel to one another, separated by a distance. Its capacitance is given by

$C = \frac{\varepsilon_0 A}{d}$ (when air is between the plates)

$C = \frac{K\varepsilon_0 A}{d}$ (when dielectric is between the plates)

where A is area of each plate and d is separation between the two plates.

When a dielectric slab of thickness t and dielectric constant K is introduced between the plates, then the capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

When a metallic conductor of thickness t is introduced between the plates, then capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

Cylindrical capacitor : It consists of two co-axial cylinders of same length.

Capacitance of an air filled cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

where a and b are the inner and outer radii and L is the length.

Spherical capacitor : It consists of two concentric spherical shells.

Capacitance of an air filled spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

where a and b are the inner and outer radii.

Illustration 10 : A sphere of radius 0.03 m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 V and outer sphere is earthed, find the capacitance and the charge on the inner sphere.

Soln.: Here a = 0.03 m, b = 0.05 m, V = 1500 V

The capacitance of the air-filled spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{0.03\times0.05}{9\times10^9\times(0.05-0.03)}$$

$= 8.33 \times 10^{-12}$ F = 8.33 pF.

Charge $q = CV = 8.33 \times 10^{-12} \times 1500$

$= 1.25 \times 10^{-8}$ C.

Grouping of Capacitors

Capacitors in series : For n capacitors connected in series, the equivalent capacitance C_S is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

Capacitors in parallel : For n capacitors connected in parallel, the equivalent capacitance C_P is given by

$C_P = C_1 + C_2 + ... + C_n$

When capacitors are connected in series, the charge through each capacitor is same.

When capacitors are connected in parallel, the potential difference across each capacitor is same.

ENERGY STORED IN A CAPACITOR

Work done in charging a capacitor gets stored in the capacitor in the form of its electric potential energy and it is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy Density

The energy stored per unit volume in the electric field between the plates is called energy density (u). It is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

SHARING OF CHARGES

When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential

till their potentials become equal. The equal potential is called common potential. It is given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

In sharing charges, there is absolutely no loss of charge. Some energy is, however, lost in the process in the form of heat etc which is given by

$$U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}.$$

Illustration 11 : Two capacitors of capacitances C_1 = 3 μF and C_2 = 6 μF arranged in series are connected in parallel with a third capacitor C_3 = 4 μF. The arrangement is connected to a 6.0 V battery. Calculate the total energy stored in the capacitors.

Soln.: Equivalent capacitance of the series combination of C_1 and C_2 is given by

$$C' = \frac{C_1C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu\text{F}$$

Combination C' is in parallel with C_3.

∴ Total capacitance,

$$C = C' + C_3 = 2 + 4 = 6\ \mu\text{F} = 6 \times 10^{-6}\ \text{F}$$

Energy stored,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 6^2$$

$$= 1.08 \times 10^{-4}\ \text{J}.$$

Effect of Dielectric

When a dielectric slab of dielectric constant K is introduced between the plates of a charged parallel plate capacitor and the charging battery remains connected, then

- Potential difference between the plates remains constant *i.e.*, $V = V_0$
- Capacitance C increases *i.e.*, $C = KC_0$
- Charge on a capacitor increases *i.e.*, $Q = KQ_0$
- Electric field between the plates remains unchanged *i.e.*, $E = E_0$
- Energy stored in a capacitor increases *i.e.*, $U = KU_0$

When a dielectric slab of dielectric constant K is introduced between the plates of a charged parallel plate capacitor and the charging battery is disconnected, then

- Charge remains unchanged *i.e.*, $Q = Q_0$
- Capacitance increases *i.e.*, $C = KC_0$
- Potential difference between the plates decreases *i.e.*, $V = \frac{V_0}{K}$
- Electric field between the plates decreases *i.e.*, $E = \frac{E_0}{K}$
- Energy stored in the capacitor decreases *i.e.*, $U = \frac{U_0}{K}$

where Q_0, C_0, V_0, E_0 and U_0 represents the charge, capacitance, potential difference, electric field and energy stored in the capacitor of a charged air filled parallel plate capacitor.

Van de Graaff generator

A Van de Graaff generator consists of a large spherical conducting shell (a few metres in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell, and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

ELECTRIC CURRENT

The time rate of flow of charge through a cross section area is called **current.**

$I_{av} = \frac{\Delta Q}{\Delta t}$ and instantaneous current $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

Direction of current is along the direction of flow of positive charge or opposite to the direction of flow of negative charge. But the current is a scalar quantity.

$$\xrightarrow{} I \qquad\qquad \xleftarrow{} I$$

$$q\oplus \longrightarrow \text{velocity} \qquad q\ominus \longrightarrow \text{velocity}$$

The SI unit of current is **ampere** and

1 ampere = 1 coloumb/second (C s^{-1})

1 coloumb/second = 1 A

Electric Current in a Conductor

In the absence of potential difference across a conductor no net current flows through a cross-section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) start drifting in a direction *i.e.* opposite to electric field with drift velocity.

If A is area of cross-section and n is number of free electrons per unit volume then,

$$I = nAev_d$$

where v_d is the drift velocity

$$v_d = \frac{eE}{m}\tau,$$

where e is electronic charge, E is electric field, m is mass of an electron, τ is relaxation time.

Illustration 12 : Find the number of free electrons per unit volume in a metallic wire of density 10^4 kg m^{-3}, atomic mass number 100 and number of free electron per atom is one.

Soln.: Number of free charge particles per unit volume

$$n = \frac{\text{Total free charge particles}}{\text{Total volume}}$$

$\therefore$ Number of free electrons per atom means total free electrons = total number of atoms

$$= \frac{N_A}{M_W} \times M$$

So $$n = \frac{\frac{N_A}{M_W} \times M}{V} = \frac{N_A}{M_W} \times \rho = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}}$$

$$n = 6.023 \times 10^{28}$$

Illustration 13 : Flow of charge through a surface is given as

$Q = 4t^2 + 2t$ (for 0 to 10 s)

(a) Find the current through the surface at $t = 5$ s.

(b) Find the average current for (0 – 10 s).

Soln.: (a) Instantaneous current

$$I = \frac{dQ}{dt} = \frac{d}{dt}(4t^2 + 2t) = 8t + 2$$

At $t = 5$ s

$$I = 8 \times 5 + 2 = 42 \text{ A}$$

(b) Average current

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t} = \frac{4 \times (10)^2 + 2 \times 10}{10} = \frac{420}{10} = 42 \text{ A}$$

CURRENT DENSITY

In a current carrying conductor we can define a vector which gives the direction as current per unit normal cross-sectional area.

Thus $\vec{J} = \frac{I}{S}\hat{n}$ or $I = \vec{J} \cdot \vec{S}$

where $\hat{n}$ is the unit vector in the direction of the flow of current.

For random J or S, we use $I = \int \vec{J} \cdot d\vec{S}$

$$\vec{J} = \frac{dI}{dS}\vec{n}$$

So $$dI = \vec{J} \cdot d\vec{S}$$

Current is the flux of current density.

Due to principle of conservation of charge,

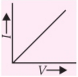
charge entering at one end of a conductor = charge leaving at the other end, so current does not change with change in cross section and conductor remains uncharged when current flows through it.

ohm's Law : The current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the temperature and other physical conditions remain unchaned. Potential difference $\propto$ current, $V \propto I$ or $V = RI$

The proportionality constant R is called the resistance of the conductor. Its value is independent of V and I but depends on the nature of the conductor, its length and area of cross-section and physical conditions like temperature, etc. Ohm's law may also be expressed as

$$\frac{V}{I} = R$$

The graph between the potential difference V applied across a conductor to the current I flowing through it is straight line, as shown in figure.



ELECTRICAL RESISTANCE

The property of a substance by virtue of that it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have $I = neAv_d = neA\left(\frac{eE}{m}\right)\tau = \left(\frac{ne^2\tau}{m}\right)AE$

If V is potential difference applied to the ends of a conductor of length l, then

$$E = \frac{V}{l}$$

So $$I = \left(\frac{ne^2\tau}{m}\right)\left(\frac{A}{l}\right)V = \left(\frac{A}{\rho l}\right)V = \frac{V}{R} \Rightarrow V = IR$$

Here, $R = \frac{A}{\rho l}$,

ρ is called **resistivity** (it is also called specific resistance),

and $\rho = \frac{m}{ne^2\tau} = \frac{1}{\sigma}$, σ is called **conductivity.** Therefore current in conductors is proportional to potential difference applied across its ends.

Units

$R \rightarrow$ ohm(Ω), $\rho \rightarrow$ ohm-meter (Ω m), $\sigma \rightarrow \Omega^{-1}$ m^{-1}.

Relation between J, E and v_d

In conductors drift velocity of electrons is proportional to the electric field inside the conductor.

As $v_d = \mu E$

where μ is the **mobility** of electrons

Current density is given as

$$J = \frac{I}{A} = nev_d = ne(\mu E) = \sigma E$$

where $\sigma = ne\mu$ is called conductivity of material and we can also write $\rho = \frac{1}{\sigma}$ = resistivity of material. Thus $E = \rho J$. It is called as **differential form of Ohm's law.**

Dependence of Resistance on various factors

$$R = \rho\frac{l}{A} = \frac{m}{ne^2\tau}\cdot\frac{l}{A}$$

Therefore, R depends as

- $\propto l$
- $\propto \frac{1}{A}$
- $\propto \frac{1}{n}$
- $\propto \frac{1}{\tau}$

In metals τ decreases as T increases, therefore R also increases.

The resistance of most conductors and all pure metals increases with temperature, but there are a few in which resistance decreases with temperature. If R_o and R be the resistance of a conductor at 0°C and at θ°C, then it is found that $R = R_o(1 + \alpha\theta)$.

where α is temperature coefficient of resistance.

The unit of α is K^{-1} or $°C^{-1}$.

Illustration 14 : If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

Soln.: As we know that, $R = \frac{\rho l}{A}$ ...(i)

In case of stretching, $R' = \frac{\rho l'}{A'}$, $l' = 2l$

Volume of the wire remains constant

$\therefore A'l' = Al$

$A' = \frac{A}{2}$ $(\because \quad l' = 2l)$

$R' = \frac{\rho \times 2l}{A/2} = 4R$ (Using (i))

ELECTRICAL POWER

The energy liberated per second in a device is called its power . The electrical power P delivered by an electrical device is given by $P = VI$, where V = potential difference across device and I = current. If the current enters the higher potential point of the device then power is consumed by it (*i.e.* acts as load). If the current enters the lower potential point then the device supplies power (*i.e.* acts as source).

Power consumed by a resistor $P = I^2R = VI = \frac{V^2}{R}$.

HEATING EFFECT OF ELECTRIC CURRENT

When a current is passed through a resistor energy is wasted in overcoming the resistances of the wire. This energy is converted into heat.

$W = VIt$ joule

$= I^2Rt$ joule $= \frac{V^2}{R}t$ joule.

Joule Law of Electrical Heating

The heat generated (in joule) when a current of I ampere flows through a resistance of R ohm for t second is given by

$$H = I^2Rt \text{ joule} = \frac{I^2Rt}{4.2} \text{ calorie.}$$

If variable current is passing through the conductor then we use for heat produced in resistance in time 0 to t is

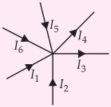
$$H = \int_0^t I^2R\,dt$$

KIRCHHOFF'S LAWS

Kirchhoff's Current Law (Junction law)

This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point of the circuit is zero" or total currents entering a junction equals total current leaving the junction. $\Sigma I_{in} = \Sigma I_{out}$. It is also known as **KCL** (Kirchhoff's current law).

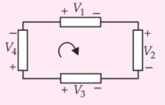
Illustration 15 : Find relation in between current I_1, I_2, I_3, I_4, I_5 and I_6.



Soln.: $I_1 + I_2 - I_3 - I_4 + I_5 + I_6 = 0$

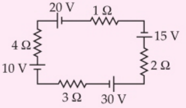
Kirchhoff's Voltage Law (Loop law)

"The algebraic sum of all the potential differences along a closed loop is zero". $\Sigma IR + \Sigma EMF = 0$". The closed loop can be traversed in any direction. While traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign.

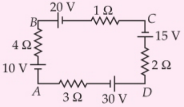


$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or nonlinear). It is also known as **KVL.** This law represents conservation of energy.

Illustration 16 : Find the current in given circuit.



Soln. : All the elements are connected in series.
$\therefore$ current in all of them will be same.
Let current = I

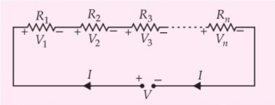


Applying Kirchhoff voltage law in $ABCDA$ loop
$10 + 4I - 20 + I + 15 + 2I - 30 + 3I = 0$
$10I = 25$
$I = 2.5$ A

COMBINATION OF RESISTANCES

A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

Resistances in Series



When the resistances are connected end to end then they are said to be in series. The current through each resistor is same. The effective resistance appearing across the battery,

$R = R_1 + R_2 + R_3 + ... + R_n$ and

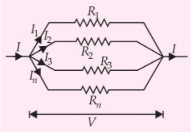
$V = V_1 + V_2 + V_3 + ... + V_n$.

The voltage across a resistor is proportional to the resistance

$$V_1 = \frac{R_1}{R_1 + R_2 + ... + R_n} V,$$

$$V_2 = \frac{R_2}{R_1 + R_2 + ... R_n} V, \text{ etc}$$

Resistances in Parallel



A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors R_1, R_2, R_3, ..., R_n.

Conclusions :

- Potential difference across each resistor is same.
- $I = I_1 + I_2 + I_3 + ... I_n$.
- Effective resistance (R) then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

- Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : .... I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : .... \frac{1}{R_n}$$

$$I_1 = \frac{G_1}{G_1 + G_2 + ... + G_n} I,$$

$$I_2 = \frac{G_2}{G_1 + G_2 + ... + G_n} I, \text{etc.}$$

where $G = \dfrac{1}{R}$ = Conductance of a resistor

EMF OF A CELL AND ITS INTERNAL RESISTANCE

Electromotive force (emf) of a cell

It is defined as the potential difference between the two terminals of a cell in an open circuit *i.e.*, when no current flows through the cell. It is denoted by symbol ε.

The SI unit of emf is joule/coulomb or volt and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

The emf of a cell depends upon the nature of electrodes, nature and the concentration of electrolyte used in the cell and its temperature.

Note : The emf is not a force.

Terminal potential difference of a cell

It is defined as the potential difference between two terminals of a cell in a closed circuit *i.e.* when current is flowing through the cell.

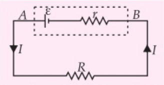
It is generally denoted by symbol V and is measured in volt.

Internal resistance of a cell

It is defined as the resistance offered by the electrolyte and electrodes of a cell when the current flows through it.

Internal resistance of a cell depends upon the following factors:

- Distance between the electrodes
- The nature of the electrolyte
- The nature of electrodes
- Area of the electrodes, immersed in the electrolyte



If a cell of emf ε and internal resistance r be connected with a resistance R, the total resistance of the circuit is $(R + r)$.

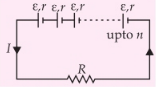
$$I = \frac{\varepsilon}{R+r},\quad V_{AB} = \frac{\varepsilon}{R+r}R$$

where

V_{AB} = Terminal voltage of the battery.

Grouping Of Cells

Cells in series



Let there be n cells each of emf ε, arranged in series. Let r be the internal resistance of each cell. Then

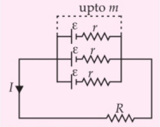
Total emf = $n\varepsilon$

Current in the circuit $I = \dfrac{n\varepsilon}{R+nr}$.

If $nr << R$ then $I = \dfrac{n\varepsilon}{R}$

If $nr >> R$ then $I = \dfrac{\varepsilon}{r}$

Cells in parallel



If m cells each of emf ε and internal resistance r be connected in parallel and if this combination be connected to an external resistance R then the emf of the circuit = ε.

Internal resistance of the circuit $= \dfrac{r}{m}$

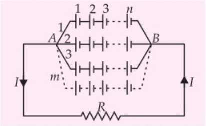
Current in circuit,

$$I = \frac{\varepsilon}{R + \frac{r}{m}} = \frac{m\varepsilon}{mR + r}$$

If $mR << r;\ I = \dfrac{m\varepsilon}{r}$

If $mR >> r;\ I = \dfrac{\varepsilon}{R}$

Cells in mixed grouping



mn = number of identical cells

n = number of cells in each rows

m = number of rows

The combination of cells is equivalent to single cell of emf = $n\varepsilon$ and internal resistance $= \dfrac{nr}{m}$

Current $I = \dfrac{n\varepsilon}{R + \frac{nr}{m}}$.

For maximum current $mR = nr$

or $\quad R = \dfrac{nr}{m}$

$$I_{\max} = \frac{m\varepsilon}{2r} = \frac{n\varepsilon}{2R}$$

Illustration 17 : Six cells are connected (a) in series, (b) in parallel and (c) in 2 rows each containing 3 cells. The emf of each cell is 1.08 V and its internal resistance is 1 ohm. Calculate the currents that would flow through an external resistance of 5 Ω in the three cases.

Soln.: (a) The cells in series.

Given that ε = 1.08 V, $n = 6$, $r = 1\ \Omega$, $R = 5\ \Omega$

The total emf = $n\varepsilon = 6 \times 1.08$ V

The total internal resistance = $nr = 6 \times 1 = 6\ \Omega$

The current in the circuit

$$I_s = \frac{n\varepsilon}{R+nr} = \frac{6\times 1.08}{5+6} = 0.589\text{ A}$$

(b) The cells in parallel,

Here ε = 1.08 V, $m = 6$, $r = 1\ \Omega$, $R = 5\ \Omega$

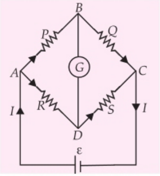
$$I_p = \frac{m\varepsilon}{mR+r} = \frac{6\times 1.08}{6\times 5+1} = \frac{6.48}{31} = 0.209\text{ A}$$

(c) The cells in mixed grouping with $n = 3$, $m = 2$

$$I = \frac{mn\varepsilon}{mR+nr} = \frac{6\times 1.08}{(2\times 5)+(3\times 1)} = \frac{6.48}{13} = 0.498\text{ A}$$

WHEATSTONE'S BRIDGE

It is an arrangement of four resistances P, Q, R and S connected as shown in the figure.

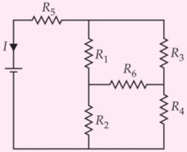


Their values are so adjusted that the galvanometer G shows no deflection. The bridge is then said to be balanced. When this happens, the points B and D are at the same potential and no current flows through galvanometer and it can be shown that

$$\frac{P}{Q}=\frac{R}{S}$$

This is called the balancing condition. If any three resistances are known, the fourth can be found.

Illustration 18 : In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\dfrac{1}{R_5}+\dfrac{1}{R_6}=\dfrac{1}{R_1+R_2}+\dfrac{1}{R_3+R_4}$

(c) $R_1R_4 = R_2R_3$ (d) $R_1R_3 = R_2R_4$

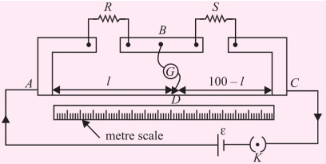
Soln.: (c) In Wheatstone's bridge, while it is balanced, no current flows through the galvanometer arm. Current is independent of galvanometer resistance. Along those lines, it can inferred that I can be independent of R_6 only when R_1, R_2, R_3 and R_4 belong to resistance arms and R_6 belongs to galvanometer arm.

In balanced bridge,

$\dfrac{R_1}{R_2}=\dfrac{R_3}{R_4}$ or $R_1R_4 = R_2R_3$.

METRE BRIDGE OR SLIDE METRE BRIDGE

It is based on the principle of Wheatstone's bridge.



Wire AC is 1 m long. R is a resistance to be measured and S is a standard resistance of known value.

The unknown resistance, $R=\dfrac{Sl}{100-l}$

where l is the balancing length of metre bridge.

POTENTIOMETER

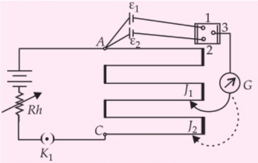
Principle of potentiometer : It is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross-section and a constant current is flowing through it.

i.e., $V \propto l$ (If I and A are constants)

or $V = Kl$

where K is known as potential gradient *i.e.*, fall of potential per unit length of the given wire.

Comparison of emfs of two cells by using potentiometer

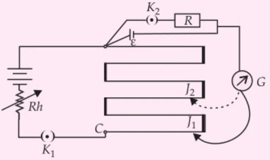


G is a galvanometer and Rh is rheostat, 1, 2, 3, are terminals of a two way key

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1}{l_2}$$

where l_1, l_2 are the balancing lengths of potentiometer wire for the emfs ε_1 and ε_2 of two cells respectively.

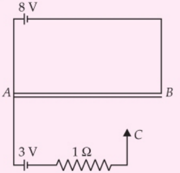
Determination of internal resistance of a cell by potentiometer



$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$

where l_1 = balancing length of potentiometer wire corresponding to emf of the cell, l_2 = balancing length of potentiometer wire corresponding to terminal potential difference of the cell when a resistance R is connected in series with the cell whose internal resistance is to be determined.

Illustration 19 : A 8 V battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 3 V and internal resistance 1 Ω is joined to the point as shown here. Take the potential at B as zero.



(a) What are the potentials at the point A and C?
(b) At which point D of the wire AB, the potential of the wire is equal to the potential at C.
(c) If the points C and D are connected by a resistor of 1 Ω, what will be the current through the resistor?

Soln.: (a) We take potential at B as 0 V. If we go to A via 8 V battery, potential at A is + 8 V.
Then we go to point C via 3 V battery. Potential drops by 3 V on crossing the 3 V battery. There is no. p.d. across 1 Ω resistor as there is no current in it.
Thus potential at C is
(+ 8 V – 3 V) = 5 V
(b) Now the p.d. across wire AB is
$V_A - V_B = 8\text{ V} - 0\text{ V} = 8\text{ V}$
The potential gradient is $\left(\frac{8\text{ V}}{100\text{ cm}}\right)$.
Let the potential at point D be the same as V_C.
$\Rightarrow V_D = 5\text{ V}$
But $V_D = \left(\frac{8\text{ V}}{100\text{ cm}}\right)x = 5\text{ V}$
$\Rightarrow x = \left(\frac{5 \times 100}{8}\right) = 62.5\text{ cm}$
Hence point D is 62.5 cm from B.
(c) There won't be any current in the resistor connecting C and D points as p.d. across it is zero *i.e.* $V_C - V_D = 0$.

QUESTIONS FOR PRACTICE

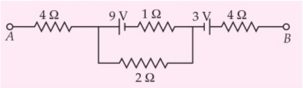
1. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is
(a) $F/4$ (b) $3F/4$ (c) $F/8$ (d) $3F/8$.

2. Two points charges A and B of values $+5 \times 10^{-9}$ C and $+3 \times 10^{-9}$ C are kept 6 cm apart in air. When the charge B is moved by 1 cm towards charge A, then work done is equal to
(a) 4.5×10^{-6} J (b) 4.5×10^{-7} J
(c) 3.5×10^{-7} J (d) 4.5×10^{-8} J

3. An electric dipole is placed at an angle of 30° with an electric field of intensity 2×10^5 N C^{-1}. It experiences a torque equal to 4 N m. Calculate the charge on the dipole if the dipole length is 2 cm.
(a) 2 mC (b) 4 mC (c) 2 μC (d) 4 μC

4. A 10 μF capacitor is charged to a potential difference of 1000 V. The terminals of the charged capacitor are disconnected from the power supply and connected to the terminals of an uncharged 6 μF capacitor. What is the final potential difference across each capacitor?
(a) 167 V (b) 100 V
(c) 625 V (d) 250 V

5. In the circuit shown in figure, potential difference between points A and B is 16 V. The current passing through 2 Ω resistance will be



(a) 2.5 A (b) 3.5 A
(c) 4.0 A (d) zero

6. n identical cells are joined in series with two cells A and B with reversed polarities. EMF of each cell is ε and internal resistance is r. Potential difference across cell A or B is ($n > 4$)
(a) $\dfrac{2\varepsilon}{n}$ (b) $2\varepsilon\left(1-\dfrac{1}{n}\right)$
(c) $\dfrac{4\varepsilon}{n}$ (d) $2\varepsilon\left(1-\dfrac{2}{n}\right)$

7. A rigid container with thermally insulated walls contains a coil of resistance 100 Ω, carrying current 1 A. Change in internal energy after 5 min will be
(a) zero (b) 10 kJ (c) 20 kJ (d) 30 kJ

8. Two spheres of radii R_1 and R_2 respectively are charged and joined by a wire. The ratio of electric field at the surface of the spheres is
(a) $\dfrac{R_2^2}{R_1^2}$ (b) $\dfrac{R_1^2}{R_2^2}$ (c) $\dfrac{R_2}{R_1}$ (d) $\dfrac{R_1}{R_2}$

9. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then, in equilibrium position, the ratio of the magnitude of electric fields at the surface of the spheres A and B is
(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1

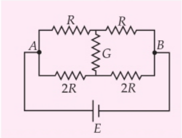
10. The sides of a rectangular block are 2 cm, 3 cm and 4 cm. The ratio of maximum to minimum resistance between its parallel faces is
(a) 4 (b) 3 (c) 2 (d) 1

11. Potentiometer wire of length 1 m is connected in series with 490 Ω resistance and 2 V battery. If 0.2 mV cm^{-1} is the potential gradient, then resistance of the potentiometer wire is
(a) 4.9 Ω (b) 7.9 Ω (c) 5.9 Ω (d) 6.9 Ω

12. A conducing wire having 10^{29} free electrons per m^3 carries a current of 20 A. If the cross-section of the wire is 1 mm^2, then the drift velocity of electrons will be (Take $e = 1.6 \times 10^{-19}$ C)
(a) 1.25×10^{-4} m s^{-1} (b) 1.25×10^{-3} m s^{-1}
(c) 1.25×10^{-5} m s^{-1} (d) 6.25×10^{-3} m s^{-1}

13. A resistor has a colour code of green, blue, brown and silver. What is its resistance?
(a) 56 Ω ± 5% (b) 560 Ω ± 10%
(c) 560 Ω ± 5% (d) 5600 Ω ± 10%

14. Consider the following statements regarding the network shown in the figure.



(1) The equivalent resistance of the network between points A and B is independent of value of G.
(2) The equivalent resistance of the network between points A and B is $\frac{4}{3}R$.
(3) The current through G is zero.

Which of the above statements is/are true?

(a) (1) only (b) (2) only
(c) (2) and (3) (d) (1), (2) and (3)

15. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply. How much electrical energy is stored by the capacitor?

(a) 2.55×10^{-6} J (b) -2.55×10^{-6} J
(c) -2.05×10^{-6} J (d) 2.05×10^{-6} J

16. The flux entering and leaving a closed surface are 5×10^5 and 4×10^5 MKS units respectively, then the charge inside the surface will be

(a) -8.85×10^{-7} C (b) 8.85×10^{-7} C
(c) 8.85×10^{7} C (d) -6.85×10^{-7} C

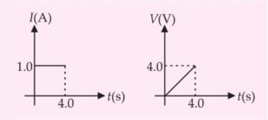
17. A conducting sphere of radius 10 cm is charged with 10 μC. Another uncharged sphere of radius 20 cm is allowed to touch it. After that if the spheres are separated, the surface density of charges on the spheres will be in the ratio of

(a) 1 : 4 (b) 1 : 2
(c) 1 : 3 (d) 2 : 1

18. A 4 m long wire of resistance 8 Ω is connected in series with a battery of emf 2 V and a resistor of 7 Ω. The internal resistance of the battery is 1 Ω. What is the potential gradient along the wire?

(a) 1.00 V m^{-1} (b) 0.75 V m^{-1}
(c) 0.50 V m^{-1} (d) 0.25 V m^{-1}

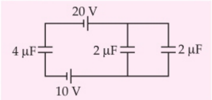
19. Current versus time and voltage versus time graph of a circuit element is shown below



The type of the circuit element is

(a) capacitor of 2 F
(b) resistor of 2 Ω
(c) capacitor of 1 F
(d) a voltage source of emf 1 V.

20. In the circuit shown in figure, charge stored in 4 μF capacitor is



(a) 20 μC (b) 40 μC (c) 30 μC (d) zero

21. A cell develops the same power across two resistances R_1 and R_2 separately. The internal resistance of the cell is

(a) $R_1 + R_2$ (b) $\frac{R_1 + R_2}{2}$
(c) $\sqrt{R_1 R_2}$ (d) $\frac{\sqrt{R_1 R_2}}{2}$

22. A voltmeter with resistance 500 Ω is used to measure the emf of a cell of internal resistance 4 Ω. The percentage error in the reading of the voltmeter will be

(a) 0.2% (b) 0.8% (c) 1.4% (d) 2.2%

Directions : Question numbers 23-24 contain Statement-1 and Statement-2. Of the four choices given choose the one that best describes the two statements.

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is true, Statement-2 is false.

23. **Statement-1:** The capacity of a conductor, under given circumstances, remains constant irrespective of the charge present on it.
Statement-2: Capacity depends on size and shape of conductor and also on the surrounding medium.

24. **Statement-1:** Current flows only when a source of p.d. is connected across a material, which has excess of free charge carriers.
Statement-2: Source of p.d. cannot produce free charge carriers. It only directs the free charge carriers to move in a particular direction.

25. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. The current in the circuit is 0.5 A. The terminal voltage of the battery when the circuit is closed, is

(a) 10 V (b) 0 V (c) 1.5 V (d) 8.5 V

26. The current in a conductor varies with time t as $I = 2t + 3t^2$ where I is in ampere and t in seconds.

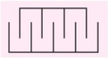
Electric charge flowing through a section of conductor during $t = 2$ s to $t = 3$ s is

(a) 10 C (b) 24 C (c) 33 C (d) 44 C

27. Two parallel large thin metal sheets have equal surface charge densities ($\sigma = 26.4 \times 10^{-12}$ C m^{-2}) of opposite signs. The electric field between these sheets is

(a) 1.5 N C^{-1} (b) 1.5×10^{-10} N C^{-1}
(c) 3 N C^{-1} (d) 3×10^{-10} N C^{-1}

28. A gang capacitor is formed by inter locking a number of plates as shown in the figure. The distance between the consecutive plates is 0.885 cm and the overlapping area of the plates is 5 cm^2. The capacity of the unit is

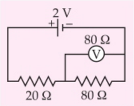


(a) 1.06 pF
(b) 4 pF
(c) 6.36 pF
(d) 12.72 pF

29. Two flash light electric incandescent lamps, each requiring 3 A at 1.5 V are placed in series and connected to a 6 V cell. What resistance must be connected in series to operate them?

(a) 22 Ω (b) 2 Ω (c) 6 Ω (d) 1 Ω

30. In the figure shown below the e.m.f. of the cell is 2 V and internal resistance is negligible. The resistance of the voltmeter is 80 Ω. The reading of voltmeter will be



(a) 2.00 V (b) 1.33 V
(c) 1.60 V (d) 0.80 V

SOLUTIONS

1. (d): Initially, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{d^2}$ ...(i)

when the third equal conductor touches B, the charge of B is shared equally between them.

$\therefore$ Charge on $B = \dfrac{q}{2}$ = charge on third conductor

Now this third conductor with charge $\left(\dfrac{q}{2}\right)$ touches C, their total charge $\left(q + \dfrac{q}{2}\right)$ is equally shared between them.

$\therefore$ Charge on $C = \dfrac{3q}{4}$ = charge on third conductor

$\therefore$ New force between B and C

$$F' = \frac{1}{4\pi\varepsilon_0 d^2}\left(\frac{q}{2} \times \frac{3q}{4}\right) = \frac{3}{8}F \quad \text{(using (i))}$$

2. (b): $q_A = +5 \times 10^{-9}$ C, $q_B = 3 \times 10^{-9}$ C

Initial distance between the charges $(r_1) = 6 \times 10^{-2}$ m

Final distance between charges $(r_2) = 5 \times 10^{-2}$ m

In moving the charge,

Work done = Final P.E. – Initial P.E.

$$= Kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= 9 \times 10^9 \times 5 \times 10^{-9} \times 3 \times 10^{-9} \times \left(\frac{1}{5 \times 10^{-2}} - \frac{1}{6 \times 10^{-2}}\right)$$

$$= 135 \times 10^{-9} \times \frac{1}{30 \times 10^{-2}}$$

$$= 4.5 \times 10^{-7}\ \text{J}$$

3. (a): Torque, $\vec{\tau} = \vec{p} \times \vec{E}$

$\therefore$ $\tau = pE\sin\theta$

or $4 = p \times 2 \times 10^5 \times \sin 30°$

or $p = \dfrac{4}{2 \times 10^5 \times \sin 30°} = 4 \times 10^{-5}$ C m

Dipole moment, $p = ql$

$\therefore$ $q = \dfrac{p}{l} = \dfrac{4 \times 10^{-5}}{0.02} = 2 \times 10^{-3}\ \text{C} = 2\ \text{mC}$

4. (c): After charging, total charge on the capacitor

$q = CV$ (where $C = 10\ \mu$F)

$\therefore$ $q = 10 \times 10^{-6} \times 1000 = 10^{-2}$ C

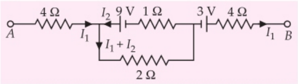
When this charged capacitor is connected to uncharged capacitor then total charge remains same.

$\therefore$ $q = q_1 + q_2$

$10^{-2} = (C_1 + C_2)V$

$\therefore$ $V = \dfrac{10^{-2}}{16 \times 10^{-6}} = 625$ V

5. (b):



$V_A - V_B = 16$ V

$\therefore$ $4I_1 + 2(I_1 + I_2) - 3 + 4I_1 = 16$ ...(i)

Using Kirchhoff's second law in the closed loop, we have

$9 - I_2 - 2(I_1 + I_2) = 0$ ...(ii)

Solving equations (i) and (ii), we get

$I_1 = 1.5$ A and $I_2 = 2$ A

$\therefore$ Current through 2 Ω resistor

$= 2 + 1.5 = 3.5$ A

6. **(d):** Current in the circuit will be $I = \frac{(n-4)\varepsilon}{nr}$.

Hence, potential difference across A or B is

$$V = \varepsilon + Ir = \varepsilon + \frac{(n-4)\varepsilon}{nr} r = 2\varepsilon\left(1 - \frac{2}{n}\right)$$

7. **(d):** As container is having thermally insulated walls. Hence, $W = 0$.

Therefore, from first law of thermodynamics,

$$\Delta U = \Delta Q = I^2Rt = (1)^2 (100) (5 \times 60) \text{ J} = 30 \text{ kJ}$$

8. **(c):** Joined by a wire means they are at the same potential. For same potential

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$

Further, the electric field at the surface of the sphere having radius R and charge q is $\frac{Kq}{R^2}$

$$\therefore \quad \frac{E_1}{E_2} = \frac{Kq_1/R_1^2}{Kq_2/R_2^2} = \left(\frac{Kq_1/R_1}{Kq_2/R_2}\right) \times \frac{R_2}{R_1} = \frac{R_2}{R_1}$$

9. **(d):** When joined by a wire, the two spheres attain common potential V.

$$\therefore \quad \text{Electric field, } E_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{R_A^2} = \frac{V}{R_A}$$

$$\text{Similarly, } E_B = \frac{V}{R_B} \quad \therefore \quad \frac{E_A}{E_B} = \frac{R_B}{R_A} = \frac{2}{1}$$

10. **(a):** $R_1 = \frac{\rho l_1}{A_1}$; $R_2 = \frac{\rho l_2}{A_2}$

$$R_1 = \frac{\rho l_1}{A_1} \times \frac{l_1}{l_1} = \frac{\rho l_1^2}{V}$$

$$R_2 = \frac{\rho l_2}{A_2} \times \frac{l_2}{l_2} = \frac{\rho l_2^2}{V} \quad \therefore \quad \frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$$

For maximum resistance, $l_1 = 4$ cm

For minimum resistance, $l_2 = 2$ cm

$$\therefore \quad \frac{R_1}{R_2} = \frac{16}{4} = 4$$

11. **(a):** Potential across potentiometer wire

$$\because \quad V = \frac{(0.2 \times 10^{-3}) \text{ V} \times 1 \text{ m}}{10^{-2} \text{ m}} = 0.02 \text{ V}$$

$$\because \quad V = \frac{R\varepsilon}{r + R}$$

$$\therefore \quad 0.02 = \frac{R}{r+R} \times 2$$

(where R is resistance of potentiometer wire and r is resistance connected in series.)

or $\quad 0.02(490 + R) = 2R$

$\Rightarrow \quad R = 4.9\ \Omega$

12. **(b):** Here, $I = 20$ A, $n = 10^{29}$ m^{-3}

$A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$, $e = 1.6 \times 10^{-19}$ C

Drift velocity, $v_d = \frac{I}{nAe}$

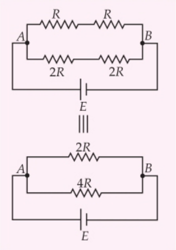
$$= \frac{20 \text{ A}}{10^{29} \text{ m}^{-3} \times 10^{-6} \text{ m}^2 \times 1.6 \times 10^{-19} \text{ C}}$$

$= 1.25 \times 10^{-3}$ m s^{-1}

13. **(b):** Numbers corresponding to green, blue, brown and silver are 5, 6, 1 and 10% respectively. Therefore, the resistance of given resistor

$= 56 \times 10^1\ \Omega \pm 10\% = 560\ \Omega \pm 10\%$

14. **(d):** As $\frac{R}{R} = \frac{2R}{2R}$, hence the given network is a balanced Wheatstone bridge. Therefore, no current flows through G, so equivalent circuit diagram is shown in the figure.



Hence, the equivalent resistance of the network between points A and B is

$$R_{eq} = \frac{(2R)(4R)}{2R + 4R} = \frac{4}{3}R$$

15. **(a):** Here, $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$

$d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m,

$V = 400$ V

$$\text{Electrical energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

$$= \frac{1}{2} \times \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} (400)^2$$

$= 2.55 \times 10^{-6}$ J

16. **(a):** Net flux leaving the surface,

$\Delta\phi = 4 \times 10^5 - 5 \times 10^5 = -10^5$ MKS

$\therefore$ Charge must be negative

$q = \phi\varepsilon_0 = -10^5 \times 8.85 \times 10^{-12}$

$= -8.85 \times 10^{-7}$ C

17. (d): Common potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{10 + 0}{4\pi\varepsilon_0(0.1 + 0.2)} = \frac{10}{4\pi\varepsilon_0(0.3)}$$

Charges after contact

$$q_1' = C_1 V = 4\pi\varepsilon_0(0.1) \times \frac{10}{4\pi\varepsilon_0(0.3)} = \frac{10}{3}\ \mu\text{C}$$

and $q_2' = C_2 V = 4\pi\varepsilon_0(0.2) \times \dfrac{10}{4\pi\varepsilon_0(0.3)} = \dfrac{20}{3}\ \mu\text{C}$

$\therefore$ The ratio of surface density of charges

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1'}{4\pi r_1^2} \times \frac{4\pi r_2^2}{q_2'} = \frac{q_1'}{q_2'}\left(\frac{r_2}{r_1}\right)^2$$

$$= \frac{10}{3} \times \frac{3}{20}\left(\frac{0.2}{0.1}\right)^2 = \frac{1}{2} \times 4 = 2:1$$

18. (d): Current in the potentiometer,

$$I = \frac{2}{8 + 7 + 1} = \frac{1}{8}\ \text{A}$$

Voltage drop across potentiometer wire

$$V = \frac{1}{8} \times 8 = 1\ \text{V}$$

$\therefore$ Potential gradient of potentiometer wire

$$\frac{V}{l} = \frac{1}{4} = 0.25\ \text{V m}^{-1}$$

19. (c) : In case of a capacitor,

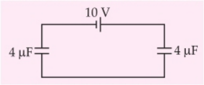
$$q = CV$$

$$\therefore \quad I = \left(\frac{dq}{dt}\right) = C\left(\frac{dV}{dt}\right)$$

As $\dfrac{dV}{dt} = \dfrac{4.0}{4.0}\ \text{V s}^{-1} = 1.0\ \text{V s}^{-1}$ (from graph)

Therefore, if $C = 1$ F
then $I = 1 \times 1 = 1$ A (constant)

20. (a) : The given circuit can be simplified as



Now potential difference across each capacitor is 10/2 or 5 V.

$\therefore \quad q = CV$
$= (4\ \mu\text{F})\ (5\ \text{V}) = 20\ \mu\text{C}$

21. (c) : Let r be the internal resistance of the cell and ε its emf. When connected across the resistance R_1 in the circuit, current passing through the resistance is

$$I = \frac{\varepsilon}{R_1 + r}$$

$$\therefore \quad P_1 = I^2 R_1 = \left(\frac{\varepsilon}{R_1 + r}\right)^2 R_1$$

Similarly $P_2 = \left(\dfrac{\varepsilon}{R_2 + r}\right)^2 R_2$

Given that $P_1 = P_2$
Substituting the values, we get

$$r = \sqrt{R_1 R_2}$$

22. (b)

23. (b): Charge present on a conductor determines its potential. Capacity is a function of size and shape of conductor and of the surrounding medium.

24. (b)

25. (d): Terminal voltage, $V = \varepsilon - Ir$
$V = 10 - 0.5 \times 3 = 10 - 1.5 = 8.5$ V

26. (b): $q = \int_2^3 I dt = \int_2^3 (2t + 3t^2)dt$

$$= [t^2 + t^3]_2^3 = 36 - 12 = 24\ \text{C}$$

27. (c) : Electric field between the sheets is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{26.4 \times 10^{-12}}{8.85 \times 10^{-12}} = 3\ \text{N C}^{-1}$$

28. (b): The given arrangement of nine plates is equivalent to the parallel combination of 8 capacitors. The capacity of each capacitor,

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 5 \times 10^{-4}}{0.885 \times 10^{-2}} = 0.5\ \text{pF}$$

The capacity of 8 capacitors = $8C = 8 \times 0.5 = 4$ pF

29. (d): Resistance of each bulb $= \dfrac{1.5}{3} = 0.5\ \Omega$
Let $R\ \Omega$ be the required series resistance. Then

$$3 = \frac{6}{R + 0.5 + 0.5}$$

$\therefore \quad R = 1\ \Omega$

30. (b): The voltmeter is in parallel with 80 Ω resistance. Let equivalent resistance be R'. Here $R' = 40\ \Omega$. Now, 20 Ω resistance is in series with R'. So, the equivalent resistance of the circuit $= 20 + 40 = 60\ \Omega$.
Current in the circuit = (2/60) A

Current across 80 Ω resistance $= \dfrac{1}{2} \times \dfrac{2}{60} = \dfrac{1}{60}$ A

$\therefore$ Reading of voltmeter $= 80 \times \dfrac{1}{60} = 1.33$ V

SAMPLE PAPER

2014

CBSE CLASS XII

BOARD EXAMINATION

1. In Coulomb's law, on what factors the value of electrostatic force constant K depends? **[1]**

2. What is non-ohmic device? Give one example. **[1]**

3. In a single slit diffraction experiment, the width of the slit is made double of the original width. How does this affect the size and intensity of the central diffraction band? **[1]**

4. Would sky waves be suitable for transmission of TV signals of 60 MHz frequency? **[1]**

5. What is the colour of the third band of a coded resistor of resistance $2.3 \times 10^2\ \Omega$? **[1]**

6. When current in a coil changes with time, how is the back e.m.f. induced in the coil related to it? **[1]**

7. Why do magnetic lines of force prefer to pass through ferromagnetic materials? **[1]**

8. Is it necessary for a transmitting antenna to be at the same height as that of the receiving antenna for the line of sight communication? A TV transmitting antenna is 81 m tall. How much service area it can cover if the receiving antenna is at the ground level? **[1]**

9. Three charges $+q$, $+2q$ and $-4q$ are placed on the vertices of an equilateral triangle. If $q = 1.0 \times 10^{-7}$ C and each side of triangle is 0.1 m, find electrostatic potential energy of the system. **[2]**

10. Explain the term stopping potential and threshold frequency. **[2]**

OR

An object is placed 18 cm in front of a mirror. If the image is formed at 4 cm to the right of the mirror, calculate its focal length. Is the mirror convex or concave? What is the nature of the image? What is the radius of curvature of the mirror? **[2]**

11. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism. **[2]**

12. A light bulb is rated at 100 W for a 220 V supply. Find
    (a) the resistance of the bulb.
    (b) the peak voltage of the source
    (c) the rms current through the bulb **[2]**

13. Two wires A and B are formed from the same material with same mass. Diameter of wire A is half of diameter of wire B. If the resistance of wire A is 32 Ω, find the resistance of wire B. **[2]**

14. What is a radial magnetic field? How has it been achieved in moving coil galvanometer? **[2]**

15. Two large parallel thin plates having uniform charge densities $+\sigma$ and $-\sigma$ are kept in X-Z plane at a distance d apart. Sketch an equipotiential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates, what is the magnitude and direction of the field? **[2]**

16. The threshold frequency of metal is υ_0. When the light of frequency $2\upsilon_0$ is incident on the metal plate, the maximum velocity of electrons emitted is v_1. When the frequency of the incident radiation is increased to $5\upsilon_0$, the maximum velocity of electrons emitted is v_2. Find the ratio of v_1 to v_2. **[2]**

17. For a common emitter amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. Suppose the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is 1 kΩ. **[2]**

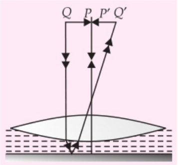
18. A proton and an alpha particle are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it, and (ii) less kinetic energy? **[2]**

19. Calculate the longest and shortest wavelength in the Balmer series of hydrogen atom. Given Rydberg constant = $1.0987 \times 10^7\ m^{-1}$. **[3]**

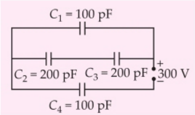
20. What is space wave propagation? Give two examples of communication system which use space wave mode.

A TV tower is 80 m tall. Calculate the maximum distance upto which the signal transmitted from the tower can be received. **[3]**

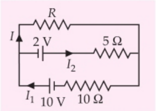
21. Figure shows an equiconvex lens of refractive index 1.5 in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of needle from the lens is measured to be 45 cm. The liquid is removed and experiment is repeated. The new distance is measured to be 30 cm. What is the refractive index of the liquid? **[3]**



22. Obtain equivalent capacitance of the following network as shown in figure. For a 300 V supply, determine the charge and voltage across each capacitor. **[3]**



23. Two cells of voltage 10 V and 2 V and internal resistances 10 Ω and 5 Ω respectively, are connected in parallel with the positive end of 10 V battery connected to negative pole of 2 V battery figure. Find the effective voltage and effective resistance of the combination. **[3]**



24. In many experimental set-ups the source and screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of the image sizes for these two points. **[3]**

25. Write the order of frequency range and one use of each of the following electromagnetic radiations

(i) Microwaves (ii) Ultra-violet rays
(iii) Gamma rays **[3]**

26. Akhil and Nikhil are arguing about the estimation of age of specimen by any scientific method. Akhil said that there is no way of finding the age of a specimen scientifically. But Nikhil argued that there should be one method to find the age of specimen, but he is not aware of that method. Tarun, who is witnessing this argument, convinced them not to proceed with the argument. He said that the age of the specimen can be estimated by noting the drop in the activity of carbon C^{14}, when the organism is dead. Listening to the explanation given by Tarun, both of them were convinced and also felt happy as they have learnt a new concept.

(i) What moral value do you observe in Tarun?
(ii) Obtain the amount of ${}_{27}Co^{60}$ necessary to provide a radioactive source of 8 mCi strength. The half life of ${}_{27}Co^{60}$ is 5.3 years. **[3]**

27. A telephonic cable at a place has four long straight horizontal wires carrying a current of 1 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G and the angle of dip is 35°. The magnetic declination is almost zero. What are the resultant magnetic fields at points 4 cm below and above the cable? **[3]**

28. Draw the energy band diagrams of p-type and n-type semiconductors. Explain with a circuit diagram the working of full-wave rectifier.

OR

Discuss common emitter amplifer, using *npn* transistor. Find its current gain, voltage gain and power gain. **[5]**

29. What are coherent source of light? State two conditions for two light source to be coherent. Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram.

OR

Describe an astronomical telescope. Derive expression for its magnifying power when final image is

(i) at infinity
(ii) at least distance of distinct vision. **[5]**

30. Explain the phenomenon of self induction. Define coefficient of self inductance. What are its units? Calculate self inductance of a long solenoid.

OR

Give the principle, construction, theory and working of an ac generator. **[5]**

SOLUTIONS

1. The value of K depends on nature of medium separating the charges and on the system of units.

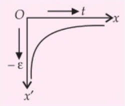
2. Non-ohmic devices are those devices which do not obey Ohm's law. Examples of non-ohmic devices are semiconductor diode, liquid electrolyte etc.

3. Width of central maximum is $2x = \frac{2D\lambda}{a}$, when width of slit (a) is doubled, width of central maxima is halved. Its area becomes $1/4^{th}$. Hence intensity of central diffraction band becomes 4 times.

4. No, because the TV signal of frequency 60 MHz is greater than the upper (limit of signal frequency of sky wave propagation *i.e.*, 40 MHz). Therefore, TV signal will not be reflected by the ionosphere but will penetrate through the ionosphere.

5. Resistance = $2.3 \times 10^2\ \Omega = 23 \times 10^1\ \Omega$. Therefore, the colour of third band of a coded resistance will be related to number 1, which is corresponding to brown colour.

6. Back emf induced in the coil, $\varepsilon = -L\frac{dI}{dt}$
where L is coefficient of self-inductance of the coil and $\frac{dI}{dt}$ is the rate of change of current through the coil. The variation of $-\varepsilon$ with t is shown in figure.



7. Magnetic lines of force prefer to pass through ferromagnetic materials because permeability (μ) and susceptibility (χ_m) of such materials are very high.

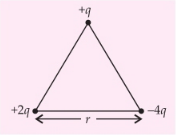
8. No, for line of sight communication, the two antenna may not be at the same height.
Service area it can cover, $A = \pi d^2 = \pi(2hR)$

$$(\because d = \sqrt{2hR})$$

$$= \frac{22}{7} \times 2 \times 81 \times 6.4 \times 10^6 = 3258.5 \times 10^6 \text{ m}^2$$

$= 3258.5 \text{ km}^2$

9. Here, $q = 1.0 \times 10^{-7}$ C and $r = 0.1$ m
Potential energy of the system of three charges,



$$U = \frac{1}{4\pi\epsilon_0 r} \cdot [q(2q) + 2q(-4q) + (-4q)q]$$

$$= \frac{-10\,q^2}{4\pi\epsilon_0 r} = -\frac{9\times10^9\times10(1.0\times10^{-7})^2}{0.1} = -9\times10^{-3}\text{ J}$$

10. Stopping potential : It is the minimum negative potential given to the anode in a photocell for which the photoelectric current becomes zero. If V_0 is the stopping potential, then maximum KE of emitted photoelectron is

$$(\text{KE})_{\text{max}} = eV_0 = h\upsilon - \phi_0$$

$$\text{or} \quad V_0 = \frac{h\upsilon}{e} - \frac{\phi_0}{e}$$

Threshold frequency: It is the minimum frequency of the incident radiation for which just emission of photoelectrons takes place from a metal surface without any KE. If υ_0 is the threshold frequency, then using Einstein's photoelectric equation,

$$0 = h\upsilon_0 - \phi_0 \quad \text{or} \quad \upsilon_0 = \frac{\phi_0}{h}$$

OR

Here, $u = -18$ cm
$v = 4$ cm

As $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

or $\frac{1}{f} = \frac{1}{4} - \frac{1}{18} = \frac{9-2}{36} = \frac{7}{36}$

$f = \frac{36}{7} = 5.14$ cm

As focal length is positive, the mirror must be convex. The image is virtual, erect and smaller in size.
Radius of curvature, $R = 2f = 2 \times 5.14$ cm
$= 10.28$ cm

11. Susceptibility of magnetic material, $\chi = \frac{I}{H}$ where I is the intensity of magnetisation induced in the material and H is the magnetising force.
Diamagnetism is due to orbital motion of electrons in an atom developing magnetic moments opposite to applied field. Due to it, the resultant magnetic moment of the diamagnetic material is zero. Hence, susceptibility χ of diamagnetic material is not much affected by temperature.

Paramagnetism and ferromagnetism is due to alignments of atomic magnetic moments in the direction of the applied field. As temperature is raised, this alignment is disturbed, resulting decrease in susceptibility of both, with increase in temperature.

12. Here, $P = 100$ W, $E_v = 220$ V,

$$\text{From,} \quad P = \frac{E_v^2}{R}, \; R = \frac{E_v^2}{P} = \frac{220 \times 220}{100} = 484\,\Omega$$

$$E_0 = \sqrt{2}\; E_v = 1.414 \times 220 = 311 \text{ V}$$

$$I_v = \frac{P}{E_v} = \frac{100}{220} = 0.45 \text{ A}$$

13. Volume of A = volume of B

$$\text{or} \quad \pi\left(\frac{D_A^2}{4}\right)l_A = \pi\left(\frac{D_B^2}{4}\right)l_B \quad \text{or} \quad \frac{l_A}{l_B} = \frac{D_B^2}{D_A^2} = 4$$

$$\text{Resistance, } R = \frac{\rho l}{A} = \frac{\rho l}{(\pi D^2/4)} \quad \text{or} \quad R \propto \frac{l}{D^2}$$

$$\therefore \quad \frac{R_B}{R_A} = \frac{l_B}{l_A} \times \frac{D_A^2}{D_B^2} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

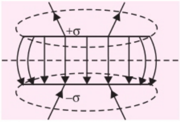
$$\text{or} \quad R_B = \frac{R_A}{16} = \frac{32}{16} = 2\,\Omega$$

14. Radial magnetic field is that field, in which the plane of the coil always lies in the direction of the magnetic field. A radial magnetic field has been achieved by

(i) properly cutting the magnetic pole pieces in the shape of concave faces

(ii) using a soft iron core within the coil.

15.



In the figure, we have shown the electric field lines by full line curves. The corresponding equipotential surfaces are shown by dotted line curves.

If E is magnitude of the field between the plates, then as particles remains stationary,

upward force due to electric field

= weight of particle

$$\text{or} \quad -qE = mg$$

$$\therefore \quad E = -\frac{mg}{q}$$

16. As υ_0 is the threshold frequency, so

Work function, $W = h\upsilon_0$

Using Einstein's photoelectric equation, we get

$$\frac{1}{2}mv_1^2 = h \times 2\upsilon_0 - W = 2h\upsilon_0 - h\upsilon_0 = h\upsilon_0 \qquad \text{...(i)}$$

$$\text{and} \quad \frac{1}{2}mv_2^2 = h \times 5\upsilon_0 - h\upsilon_0 = 5h\upsilon_0 - h\upsilon_0 = 4h\upsilon_0 \qquad \text{...(ii)}$$

Divide (i) by (ii) we get

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{h\upsilon_0}{4h\upsilon_0}$$

$$\text{or} \quad \frac{v_1}{v_2} = \frac{1}{2} = 1:2$$

17. Here, $R_0 = 2000\,\Omega$, $V_0 = 2$ V, $\beta_{ac} = 100$, $R_i = 1000\,\Omega$

$$\text{As,} \quad A_V = \frac{V_0}{V_i} = \beta_{ac}\frac{R_0}{R_i}$$

$$\text{or} \quad V_i = \frac{V_0}{\beta_{ac}.(R_0/R_i)} = \frac{2}{100(2000/1000)} = 0.01 \text{ V}$$

$$I_B = \frac{V_i}{R_i} = \frac{0.01 \text{ V}}{1000\,\Omega} = 10\,\mu\text{A}$$

18. (i) When a charged particle of charge q mass m is accelerated under a potential difference V, let v be the velocity acquired by the particle, then

$$qV = \frac{1}{2}mv^2 \quad \text{or} \quad mv = [2mqV]^{1/2}$$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\text{Hence,} \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m}{m} \times \frac{2e}{e}}$$

$$= 2\sqrt{2} > 1$$

or $\quad \lambda_p > \lambda_\alpha$,

i.e., de-Broglie wavelength associated with proton is greater than that of alpha particle.

(ii) Kinetic energy of charged particle

$$E_k = qV, \; i.e., \; E_k \propto q$$

$$\therefore \quad \frac{E_{k_p}}{E_{k_\alpha}} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2} < 1 \quad \text{or} \quad E_{k_p} < E_{k_\alpha}$$

i.e., proton has less kinetic energy than α-particle.

19. The wavelength (λ) of different spectral lines of Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad \text{where } n = 3, 4, 5, 6, \ldots\ldots$$

For longest wavelength, $n = 3$

$$\therefore \quad \frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{5 \times 1.097 \times 10^7} \text{ m}$$

$$\lambda = \frac{36 \times 10^{10}}{5 \times 1.097 \times 10^7} \text{ Å}$$

$$\lambda = 6563 \text{ Å}$$

For shortest wavelength, $n = \infty$

$$\therefore \quad \frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{1.097 \times 10^7}{4}$$

$$\Rightarrow \lambda = \frac{4}{1.097 \times 10^7} \text{ m} = \frac{4 \times 10^{10} \text{ Å}}{1.097 \times 10^7} = 3646 \text{ Å}$$

20. Space wave propagation: It is the mode of propagation in which waves travel in space in a straight line from the transmitting antenna to the receiving antenna.

Television broadcast, satellite communication are the two examples of communication system which use space waves.

Here, $h = 80$ m, $R = 6.4 \times 10^6$ m $= 64 \times 10^5$ m

$$d = \sqrt{2hR}$$

$$d = \sqrt{2 \times 80 \times 64 \times 10^5}$$

$$d = 32 \times 10^3 \text{ m} = 32 \text{ km}$$

21. Let focal length of convex lens of glass, $f_1 = 30$ cm, focal length of planoconcave lens of liquid, f_2

Combined focal length, $F = 45$ cm

$$\text{As} \quad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

$$\therefore \quad \frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{45} - \frac{1}{30} = -\frac{1}{90}$$

$$f_2 = -90 \text{ cm}$$

$$\text{As} \quad \frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For glass lens, let $R_1 = R$, $R_2 = -R$

$\mu = 3/2$

$$\therefore \quad \frac{1}{30} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{2} \times \frac{2}{R} = \frac{1}{R}$$

$R = 30$ cm

For liquid lens, $R_1 = -R = -30$ cm, $R_2 = \infty$

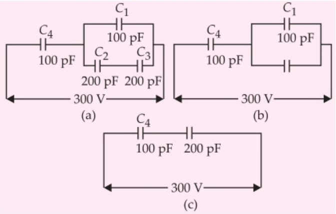
$$\text{As} \quad \frac{1}{f_2} = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu_l - 1)\left(\frac{1}{-30} - \frac{1}{\infty}\right)$$

$$\text{or} \quad \frac{1}{-90} = (\mu_l - 1) \times \frac{1}{-30}$$

$$\mu_l - 1 = \frac{30}{90} = \frac{1}{3}$$

$$\mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

22. The equivalent circuits of network in figure are shown.



In figure as 100 pF and 200 pF capacitors are in series, potential difference across C_4 is in the ratio 2 : 1.

$$\therefore \quad V_4 = \frac{300 \times 2}{2 + 1} = 200 \text{ V}$$

$$q_4 = C_4V_4 = (100 \times 10^{-12}) \times 200 = 2 \times 10^{-8} \text{ C}$$

Potential difference across 200 pF = 300 – 200 = 100 V

This must be the potential difference across C_1

i.e., $V_1 = 100$ V

$$\therefore \quad q_1 = C_1V_1 = (100 \times 10^{-12}) \times 100 = 10^{-8} \text{ C}$$

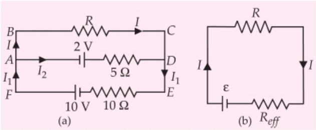
Also, potential difference across C_2, C_3 in series = 100 V

$$\therefore \quad V_2 = V_3 = \frac{100}{2} = 50 \text{ V}$$

$$q_2 = C_2V_2 = (200 \times 10^{-12}) \times 50 = 10^{-8} \text{ C},$$

$$q_3 = C_3V_3 = (200 \times 10^{-12}) \times 50 = 10^{-8} \text{ C}$$

23.



According to Kirchhoff's first rule, at junction A

$$I_1 = I + I_2$$

$$\text{or} \quad I = I_1 - I_2 \qquad \ldots\text{(i)}$$

According to Kirchhoff's second rule, in closed loop $ABCDA$, we have

$$IR - I_2 \times 5 + 2 = 0 \quad \text{or} \quad I_2 = \frac{IR+2}{5} \qquad \text{....(ii)}$$

In closed loop $BCEFB$

$$IR + I_1 \times 10 - 10 = 0 \quad \text{or} \quad I_1 = \frac{10 - IR}{10} \qquad \text{....(iii)}$$

Putting values in (i), we get

$$I = \frac{10-IR}{10} - \frac{IR+2}{5} = \frac{10 - IR - 2IR - 4}{10} = \frac{6-3IR}{10}$$

or $\quad 10I = 6 - 3IR$ or $I(10+3R) = 6$

$$\text{or} \quad I = \frac{6}{10+3R} = \frac{6/3}{(10/3)+R} \qquad \text{....(iv)}$$

If ε and R_{eff} are the effective voltage and effective internal resistance of the combination, then from figure (b).

$$\therefore \quad I = \frac{\varepsilon}{R_{eff} + R} \qquad \text{....(v)}$$

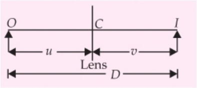
Comparing (iv) and (v), we get,

$$\varepsilon = \frac{6}{3} = 2\text{ V},$$

$$R_{eff} = \frac{10}{3}\ \Omega.$$

24. In the lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ ....(i)

we find that u and v are reversible. Therefore, there are two positions of the object, for which there will be an image on the screen.



Let the first position be when lens is at C,

$\therefore \quad -u + v = D$ = distance between source (O) and screen I.

or $u = -(D - v)$

Put in (i)

$$\frac{1}{v} + \frac{1}{D-v} = \frac{1}{f} \quad \text{or} \quad \frac{D - v + v}{v(D-v)} = \frac{1}{f}$$

or $\quad Dv - v^2 = Df$ or $v^2 - Dv + Df = 0$

$$\therefore \quad v = \frac{D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D}{2} \pm \frac{\sqrt{D^2-4Df}}{2}$$

$$\text{and} \quad u = -(D - v) = -\left[\frac{D}{2} \pm \frac{\sqrt{D^2-4Df}}{2}\right]$$

Thus if object distance is, $u = \frac{D}{2} + \frac{\sqrt{D^2-4Df}}{2}$,

then image distance is, $v = \frac{D}{2} - \frac{\sqrt{D^2-4Df}}{2}$

Again, if object distance is, $u = \frac{D}{2} - \frac{\sqrt{D^2-4Df}}{2}$,

then image distance is, $v = \frac{D}{2} + \frac{\sqrt{D^2-4Df}}{2}$

The distance between two positions of the lens for these two object distance is

$$d = \frac{D}{2} + \frac{\sqrt{D^2-4Df}}{2} - \frac{D}{2} + \frac{\sqrt{D^2-4Df}}{2} = \sqrt{D^2-4Df}$$

If $u = \frac{D}{2} + \frac{d}{2}$, then $v = \frac{D}{2} - \frac{d}{2}$,

$\therefore \quad$ Magnification, $m_1 = \frac{v}{u} = \frac{D-d}{D+d}$

If $u = \frac{D}{2} - \frac{d}{2}$, then $v = \frac{D}{2} + \frac{d}{2}$

$\therefore \quad$ Magnification, $m_2 = \frac{v}{u} = \frac{D+d}{D-d}$

25. (i) Microwaves : Order of frequency range 10^9 Hz to 10^{12} Hz.

Use : Microwaves are used for radar systems in aircraft navigation.

(ii) Ultra-violet rays : Order of frequency range 10^{14} Hz to 10^{17} Hz.

Use : Ultra-violet rays are used to destroy the bacteria and for sterilizing the surgical instruments.

(iii) Gamma rays : Order of frequency range 10^{18} Hz to 10^{22} Hz.

Use : Gamma rays are used in the treatment of cancer and tumours.

26. (i) Readiness to teach his juniors, concern of juniors towards learning.

(ii) Strength of source,

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$
$$= 8.0 \times 3.7 \times 10^7 \text{ disintegration per second}$$

Half Life, $T_{1/2}$= 5.3 years

$= 5.3 \times 365 \times 24 \times 60 \times 60$ sec $= 1.67 \times 10^8$ s

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^8} = 4.14 \times 10^{-9}\text{ s}^{-1}$$

$$\text{As} \quad \frac{dN}{dt} = \lambda N$$

$$\therefore \quad N = \frac{dN/dt}{\lambda} = \frac{8 \times 3.7 \times 10^7}{4.14 \times 10^{-9}} = 7.15 \times 10^{16}$$

By definition of Avogadro's number,

Mass of 6.023×10^{23} atoms of ${}_{27}Co^{60} = 60$ g

Mass of 7.15×10^{16} atoms of ${}_{27}Co^{60}$

$$= \frac{60 \times 7.15 \times 10^{16}}{6.023 \times 10^{23}} = 7.12 \times 10^{-6} \text{ g}$$

27. Here, no. of wires, $n = 4$, $I = 1.0$ A,
earth's field, $R = 0.39 \times 10^{-4}$ T [$\because$ 1 $G = 10^{-4}$T]
Dip angle, $\delta = 35°$, declination $\theta = 0°$
$r = 4$ cm each $= 4 \times 10^{-2}$ m
Magnetic field at 4 cm due to currents in 4 wires,

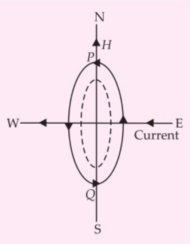
$$B = 4 \times \frac{\mu_0 I}{2\pi r} = 4 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 4 \times 10^{-2}} = 2 \times 10^{-5} \text{ T}$$

Horizontal component of earth's field

$H = R \cos\delta = 0.39 \times 10^{-4} \cos 35°$
$= 0.39 \times 10^{-4} \times 0.8192 = 3.19 \times 10^{-5}$ T

Vertical component of earth's field

$V = R \sin\delta = 0.39 \times 10^{-4} \sin 35°$
$= 0.39 \times 10^{-4} \times 0.5736 = 2.2 \times 10^{-5}$ T



At point Q, 4 cm below the wire, horizontal component due to earth's field and field due to current are in opposite directions, so that

$H_1 = H - B$

$\therefore \quad H_1 = 3.19 \times 10^{-5} - 2 \times 10^{-5} = 1.19 \times 10^{-5}$ T

Hence,

$$R_1 = \sqrt{H_1^2 + V^2}$$
$$= \sqrt{(1.19 \times 10^{-5})^2 + (2.2 \times 10^{-5})^2}$$
$$= 2.5 \times 10^{-5} \text{ T}$$

At point P, 4 cm above the wire, horizontal component of earth's field and field due to current are in the same direction,

$\therefore H_2 = H + B = 3.19 \times 10^{-5} + 2 \times 10^{-5} = 5.19 \times 10^{-5}$ T

$$R_2 = \sqrt{H_2^2 + V^2} = \sqrt{(5.19 \times 10^{-5})^2 + (2.2 \times 10^{-5})^2}$$
$$= 5.64 \times 10^{-5} \text{ T}$$

28. Refer point 9.2, 2 (a,b), 6(ii) page no. 531, 534 (MTG Excel in Physics)

OR

Refer point 9.4(8) page no. 540 (MTG Excel in Physics).

29. Refer point 6.13 page no. 406 (MTG Excel in Physics).

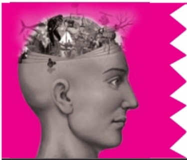
OR

Refer point 6.9(3) page no. 347 (MTG Excel in Physics).

30. Refer point 4.2(1) page no. 226 (MTG Excel in Physics).

OR

Refer point 4.8(2) page no. 252 (MTG Excel in Physics).



EXAMINER'S MIND NCERT CLASS XII

The questions given in this column have been prepared strictly on the basis of NCERT Physics for Class XII. This year JEE (Main & Advanced) / NEET/ AIIMS/other PMTs have drawn their papers heavily from NCERT books.

SEMICONDUCTOR ELECTRONICS : MATERIALS, DEVICES AND SIMPLE CIRCUITS | COMMUNICATION SYSTEMS

SECTION-1

Only One Option Correct Type

This section contains 15 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

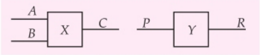
1. What would be the percentage power saving when the carrier and one of the side bands were suppressed in an AM wave modulated to a depth of 75%?
(a) 89.1% (b) 9.1%
(c) 7.81% (d) 100%

2. A ground receiver station is receiving a signal at 5 MHz, transmitted from a ground transmitter at a height of 300 m located at a distance of 100 km. Identify the mode of propagation. (Radius of earth = 6.4×10^6 m; N_{max} of ionosphere = 10^{12} m^{-3})
(a) ground wave propagation
(b) space wave propagation
(c) sky wave propagation
(d) satellite communication

3. The saturation current of a p-n junction diode made from germanium at 27°C is 10^{-5} A. What will be the required potential in order to obtain a current of 250 mA in forward bias? (Boltzman constant $k = 1.38 \times 10^{-23}$ J K^{-1})
(a) 0.26 V (b) 2.6 V
(c) 0.52 V (d) 1.04 V

4. When a p-n junction diode made from germanium or silicon is forward-biased, energy is released at the junction due to the recombination of electrons and holes. This energy is in
(a) Visible region
(b) Infrared region
(c) UV region
(d) X-ray region

5. Logic gates X and Y have the truth tables shown below

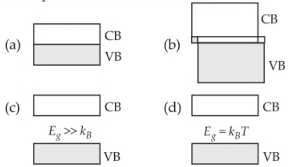


A	B	C	P	R
0	0	0	0	1
1	0	0	1	0
0	1	0	–	–
1	1	1	–	–

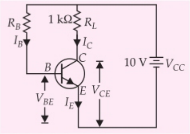
When the output of X is connected to the input of Y, the resulting combination is equivalent to a single
(a) NOT gate (b) OR gate
(c) NOR gate (d) NAND gate

6. A transistor is operated in common-emitter configuration at constant collector voltage, $V_C = 1.5$ V such that a change in the base current from 100 μA to 150 μA produces a change in the collector current from 5 mA to 100 mA. The current gain is
(a) 67 (b) 75 (c) 100 (d) 50

7. Which of the energy band diagrams shown below corresponds to that of a semiconductor?



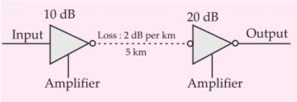
8. In the circuit shown in figure, the current gain, $\beta = 100$ for the transistor. What would be the base resistance R_B so that $V_{CE} = 5$ V? (Neglect V_{BE}).



(a) $2 \times 10^3\ \Omega$ (b) $2 \times 10^5\ \Omega$
(c) $1 \times 10^6\ \Omega$ (d) $500\ \Omega$

9. A transmitting antenna at the top of a tower has a height of 36 m and the height of the receiving antenna is 49 m. What is the maximum distance between them for satisfactory communication in LOS mode? (Radius of Earth = 6400 km).
(a) 20 km (b) 25.6 km
(c) 46.5 km (d) 68.4 km

10. Figure shows a communication system. What is the output power when input signal is of 1.01 mW? [Gain in dB = $10 \log_{10} (P_0/P_i)$]



(a) 90 mW (b) 202 mW
(c) 101 mW (d) 405 mW

11. A sky wave with a frequency 55 MHz is incident on the *D* region of Earth's atmosphere at 30°. The angle of refraction is (electron density for *D*-region is 400×10^6 electrons per m^3)
(a) 60° (b) 15° (c) 45° (d) 30°

12. A light beam entering an optical fiber makes an angle of 10° with the fiber core-fiber clad boundary surface. If the fiber core and clad refractive indices are 1.5 and 1.49 respectively, can this beam propagate along the fiber?
(a) No
(b) Yes
(c) Refractive indices have nothing to do with beam propagation
(d) The given data is insufficient

13. The length of germanium rod is 0.928 cm and its area of cross-section is 1 mm^2. If for germanium $n_i = 2.5 \times 10^{19}\ m^{-3}$, $\mu_h = 0.19\ m^2\ V^{-1}s^{-1}$ and $\mu_e = 0.39\ m^2\ V^{-1}s^{-1}$, then the resistance of rod is
(a) 2.5 kΩ (b) 4 kΩ
(c) 5 kΩ (d) 10 kΩ

14. A TV tower has a height of 100 m. What is the maximum distance upto which the TV transmission can be received ($R = 8 \times 10^6$ m)?
(a) 34.77 km (b) 32.70 km
(c) 40 km (d) 40.70 km

15. A common emitter amplifier has a voltage gain of 50, an input impedance of 100 Ω and an output impedance of 200 Ω. The power gain of the amplifier is
(a) 500 (b) 1000
(c) 1250 (d) 100

SECTION - 2

One or More Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which ONE or MORE are correct.

16. In an *npn* transistor circuit the collector current is 10 mA. If 90% of the electrons are able to reach the collector
(a) the emitter current will be 9 mA
(b) the emitter current will be 11 mA
(c) the base current will be 1 mA
(d) the base current will be 0.1 mA.

17. Which of the following statements is/are correct?
(a) The sky waves are the radiowaves of frequency upto 30 MHz
(b) The sky waves are reflected from ionosphere
(c) In a single reflection from ionosphere, the sky waves cover a distance not more than 4000 km
(d) The critical frequency for sky wave is 4 MHz for *F*-layer of ionosphere.

18. An audio signal of 15 kHz frequency cannot be transmitted over long distances without modulation because
(a) the size of the required antenna would be at least 5 km which is not convenient.
(b) the audio signal cannot be transmitted through sky waves
(c) the size of the required antenna would be at least 20 km, which is not convenient
(d) effective power transmitted would be very low, if the size of the antenna is less than 5 km

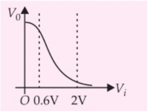
19. In amplitude modulation, the modulation index μ, is kept less than or equal to 1 because
(a) μ > 1, will result in interference between carrier frequency and message frequency, resulting into distortion
(b) μ > 1, will result in overlapping of both side bands resulting into loss of information
(c) μ > 1, will result in change in phase between carrier signal and message signal

(d) $\mu > 1$, indicates amplitude of message signal greater than amplitude of carrier signal resulting into distortion

20. Audio sine waves of 3 kHz frequency are used to amplitude modulate a carrier signal of 1.5 MHz. Which of the following statements are true?
(a) The side band frequencies are 1506 kHz and 1494 kHz.
(b) The bandwidth required for amplitude modulation is 6 kHz.
(c) The bandwidth required for amplitude modulation is 3 MHz.
(d) The side band frequencies are 1503 kHz and 1497 kHz.

21. To reduce the ripples in a rectifier circuit with capacitor filter
(a) R_L should be increased
(b) input frequency should be decreased
(c) input frequency should be increased
(d) capacitors with high capacitance should be used

22. Figure shows the transfer characteristics of a base biased *CE* transistor. Which of the following statements are true?



(a) At $V_i = 0.4$ V, transistor is in active state.
(b) At $V_i = 1$ V, it can be used as an amplifier.
(c) At $V_i = 0.5$ V, it can be used as a switch turned off.
(d) At $V_i = 2.5$ V, it can be used as a switch turned on.

23. The light emitting diode (LED)
(a) emits light when reverse biased
(b) emits light when forward biased
(c) is made from one of the two basic semiconducting materials silicon and germanium
(d) is made from semiconducting compound gallium arsenide-phosphide.

24. What happens during regulation action of a Zener diode?
(a) The current in and voltage across the Zener remains fixed.
(b) The current through the series resistance changes.
(c) The Zener resistance is constant.
(d) The resistance offered by the Zener diode changes.

25. The breakdown in a reverse biased *p-n* junction diode is more likely to occur due to
(a) large velocity of the minority charge carriers if the doping concentration is small
(b) large velocity of the minority charge carriers if the doping concentration is large
(c) strong electric field in a depletion region if the doping concentration is small
(d) strong electric field in the depletion region if the doping concentration is large

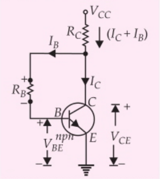
SECTION - 3

Paragraph Type

This section contains 2 paragraphs each describing theory, experiment, data, etc. Four questions relate to two paragraphs with two questions on each paragraph. Each question of a paragraph has only one correct answer among the four choices (a), (b), (c) and (d).

Paragraph for Questions 26 and 27

Figure shows an *n-p-n* transistor connected to two resistors R_B and R_C and a battery V_{CC}. I_B and I_C are the base and collector currents where $I_B << I_C$ and $\beta = I_C/I_B$ (current gain of a transistor in *CE* configuration). Take $V_{CC} = 10$ V, $R_B = 500$ kΩ, $R_C = 500\ \Omega$, $\beta = 100$.

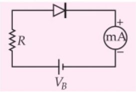


26. The base current (I_B) is
(a) 8 μA (b) 18 mA
(c) 18.2 μA (d) 8 mA

27. The collector voltage (V_C) is
(a) 18 V (b) 1.8 V
(c) 10 V (d) 9.1 V

Paragraph for Questions 28 and 29

A silicon *p-n* junction diode is connected to a resistor *R* and a battery of voltage V_B through milliammeter (mA) as shown in Figure.



The knee voltage for this junction diode is $V_N = 0.7$ V. The *p-n* junction diode requires a minimum current of 1 mA to attain a value higher than the knee point on the *I-V* characteristics of this junction diode. Assuming that the voltage *V* across the junction is independent of the current above the knee point.

28. If V_B = 5 V, the maximum value of R so that the voltage V above the knee point voltage is
(a) 40 kΩ (b) 4.3 kΩ
(c) 5.0 kΩ (d) 8.7 kΩ

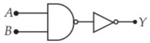
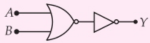
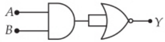
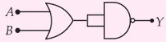
29. If V_B = 5 V, the value of R in order to establish a current of 6 mA in the circuit is
(a) 833 Ω (b) 717 Ω
(c) 950 Ω (d) 733 Ω

SECTION - 4

Matching List Type

This section contains 2 multiple choice questions. Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

30. The logic circuits are given in List I and the Boolean expressions in List II. Select the correct answer using the code given below the lists.

List I		List II	
P.		1.	$Y = \bar{A} + \bar{B}$
Q.		2.	$Y = \bar{A} \cdot \bar{B}$
R.		3.	$Y = A \cdot B$
S.		4.	$Y = A + B$

Codes

	P	Q	R	S
(a)	3	4	1	2
(b)	3	4	2	1
(c)	1	4	3	2
(d)	4	3	2	1

31. Match List I and List II and select the correct match out of the four given codes.

List I		List II	
P.	Microphone	1.	It converts pressure variation into electrical signals.
Q.	Piezo-electric senser	2.	It is the loss of strength of signal during its propagation through communication channel.
R.	Photo-detector	3.	It converts speech signal into electrical signals.
S.	Attenuation	4.	It converts light signals into electrical signals.

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	3	4	2	1
(c)	3	4	1	2
(d)	3	1	4	2

SECTION - 5

Assertion-Reason Type

This section contains 4 questions. Read the two statements in the following questions. Of the four choices given, choose the one that best describes the two statements.

(a) Statement-I is true, Statement-II is true; Statement-I is a correct explanation of Statement-I.
(b) Statement-I is true, Statement-II is true; Statement-II is not a correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is false.
(d) Statement-I is false, Statement-II is true.

32. **Statement-I :** FM broadcast is preferred over AM broadcast.
Statement-II : Process of combining the message signals with carrier wave is called demodulation.

33. **Statement-I :** Higher the modulation index, the reception will be strong and clear.
Statement-II : The degree, to which the carrier wave is modulated is called modulation index.

34. **Statement-I :** If forward current changed by 1.5 mA when forward voltage in semiconductor triode is changed from 0.5 V to 2 V, the forward resistance of diode will be 1 Ω.
Statement-II : The forward resistance is given by

$$R_f = \frac{\Delta V_f}{\Delta I_f}.$$

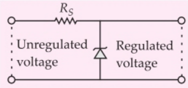
35. **Statement-I :** The surface wave propagation is used for medium frequency and low frequency band.
Statement-II : The attenuation of surface waves increases with increase in frequency.

SECTION - 6

Integer Value Correct Type

This section contains 5 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

36. A Zener of power rating 1 W is to be used as a voltage regulator. If Zener has a breakdown of 5 V and it has to regulate voltage which fluctuated between 3 V and 6 V, what should be the value of R_S for safe operation.



37. A signal wave of frequency 4.5 kHz is modulated with a carrier wave of frequency 3.45 MHz. The band width of FM wave in kHz is

38. For an amplitude modulated wave, the maximum amplitude is found to be 10 V while the minimum amplitude is found to be 0 V. Determine the modulation index μ.

39. The mean free path of conduction electrons in copper is about 4×10^{-8} m. The electric field which can give on the average 2.4 eV energy to conduction electron in copper block is $a \times 10^7$ V m^{-1}. What is the integer value of a?

40. A common emitter amplifier gives an output of 3 V for an input of 0.01 V. If β of the transistor is 100 and the input resistance is 1 kΩ, then the collector resistance in kΩ is

SOLUTIONS

1. **(a) :** Here, modulation index, $\mu = \frac{75}{100} = \frac{3}{4}$,

Power produced by the AM transmitter

As $P_t = P_c\left(1+\frac{\mu^2}{2}\right) = P_c\left(1+\frac{9}{32}\right) = P_c \times \frac{41}{32}$

$$P_{SB} = P_c\frac{\mu^2}{4} = P_c \times \frac{9}{16\times 4} = \frac{9}{64}P_c$$

On suppressing, power saved is

$$= P_c + \frac{9}{64}P_c = \frac{73}{64}P_c$$

∴ Percentage saving $= \frac{\frac{73}{64}P_c}{P_t} \times 100$

$$= \frac{\frac{73}{64}P_c}{\frac{41}{32}P_c} \times 100 = \frac{73\times 32}{64\times 41}\times 100 = 89.1\%$$

2. **(c) :** Maximum distance covered by space wave communication

$= \sqrt{2Rh_T} = \sqrt{2\times 6.4\times 10^6 \times 300} \simeq 62 \times 10^3$ m.

Since receiver-transmitter distance is 100 km. Therefore signal cannot go either by ground wave or space wave.

For sky wave, the value of critical frequency of signal should be

$$\upsilon_c = 9(N\max)^{\frac{1}{2}} = 9(10^{12})^{\frac{1}{2}} = 9\times 10^6 \text{ Hz} = 9 \text{ MHz}.$$

As the signal frequency 5 MHz < 9 MHz, so the communication is through sky waves.

3. **(a) :** Here, $I_0 = 10^{-5}$ A, $T = 27 + 273 = 300$ K,
$I = 250 \times 10^{-3}$ A

As $I = I_0[e^{eV/kT - 1}]$

$$\Rightarrow \quad e^{eV/kT} = \frac{I}{I_0} + 1 = \frac{250\times 10^{-3}}{10^{-5}} + 1 = 25001$$

or $\frac{eV}{kT} = \ln(25001) = 10.127$ or $V = \frac{10.127\times kT}{e}$ V

$$= \frac{10.127\times 1.38\times 10^{-23}\times 300}{1.6\times 10^{-19}} = 0.26 \text{ V}.$$

4. **(b) :** If p-n junction diode is made from germanium or silicon, the energy released due to recombination of free electrons and holes is in the infrared region. However, if the p-n junction diode made from gallium arsenide or indium phosphide, then the energy released due to recombination of free electrons and holes is in the visible region.

5. **(d):** The truth table of the resulting logic circuit by connecting X to Y is as follows

A	B	C	R
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Hence, form the truth table, the combination is equivalent to a single NAND gate.

6. **(c) :** As $\Delta I_B = 150\ \mu\text{A} - 100\ \mu\text{A} = 50\ \mu\text{A}$;
and $\Delta I_C = 10 \text{ mA} - 5 \text{ mA} = 5$ mA,
Current gain,

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{5 \text{ mA}}{50\ \mu\text{A}} = \frac{5\times 10^{-3}\text{ A}}{50\times 10^{-6}\text{ A}} = 100$$

7. **(d):** In a semiconductor, the energy gap (E_g) between valence band and conduction band corresponds to thermal energy (k_BT) at room temperature (T).
Hence option (d) represent the correct diagram.

8. **(b):** Here, $\beta = 100$, $V_{CE} = 5$ V, $V_{CC} = 10$ V

As $\beta = \frac{I_C}{I_B}$ or $I_B = \frac{I_C}{\beta} = \frac{I_C}{100}$ ...(i)

Also, $V_{CE} = V_{CC} - I_CR_L$

or $5 \text{ V} = 10 \text{ V} - I_C \times 1000$

$$\therefore \quad I_C = \frac{5\text{ V}}{1000\ \Omega} = 5\times 10^{-3}\text{ A}$$

and $I_B = \frac{5\times 10^{-3}\text{ A}}{100} = 5\times 10^{-5}$ A (Using (i))

Thus, $R_B = \frac{V_{CC}-V_{BE}}{I_B}$

$= \frac{10\text{ V}}{5\times 10^{-5}\text{A}} = 2\times 10^5\ \Omega$ (neglecting V_{BE})

9. **(c) :** Here, $h_T = 36$ m, $h_R = 49$ m and $R = 6400$ km
If d_M is the maximum distance between two antennas for satisfactory communication in LOS mode, then

$$d_M = d_T + d_R$$
$$= \sqrt{2Rh_T} + \sqrt{2Rh_R}$$
$$= \sqrt{(2 \times 6400 \text{ km} \times 36 \times 10^{-3} \text{km})} + \sqrt{(2 \times 6400 \text{ km} \times 49 \times 10^{-3} \text{km})}$$
$$= 21.5 \text{ km} + 25 \text{ km} = 46.5 \text{ km}$$

10. **(c) :** Total gain of both the amplifiers
$= 10$ dB $+ 20$ dB $= 30$ dB
Loss suffered in transmission path
$= (2$ dB/km$)(5$ km$) = 10$ dB
overall gain of the signal = net gain of the amplifier
$= 30$ dB $- 10$ dB $= 20$ dB
As gain in dB $= 10\log_{10}(P_0/P_i)$,
$20 = 10\log_{10}(P_0/P_i)$ or $\log_{10}(P_0/P_i) = 2$
or $(P_0/P_i) = 10^2$
or $P_0 = 10^2 P_i = 10^2 \times 1.01$ mW $= 101$ mW

11. **(d):** Here for D region, $\upsilon = 55 \times 10^6$ Hz, $i = 30°$, $N = 400 \times 10^6$ electrons per m^3

As $$\mu = \sqrt{1 - \frac{81.45\, N}{\upsilon^2}} = \sqrt{1 - \frac{81.45 \times 400 \times 10^6}{(55 \times 10^6)^2}} \simeq 1$$

Now $$\mu = \frac{\sin i}{\sin r} \text{ or } \frac{\sin i}{\sin r} = 1$$

or $\sin i = \sin r$
$\therefore$ $i = r = 30°$.

12. **(a) :** Critical angle of core-cladding surface is

$$\sin C = \frac{\mu_2}{\mu_1} = \frac{1.49}{1.50} = 0.9933 = \sin 83°21'$$

or $C = 83°21'$.
For optical fiber,

$$\frac{\sin\theta_a}{\sin(90 - \theta_i)} = \frac{\mu_1}{\mu_0} \text{ or } \frac{\sin\theta_a}{\cos\theta_i} = \frac{\mu_1}{\mu_0}$$

or $$\cos\theta_i = \frac{\mu_0}{\mu_1}\sin\theta_a = \frac{1}{1.5}\sin 10°$$
$$= \frac{2}{3} \times 0.1736 = 0.1157$$

$\theta_i = 83°23'$
As the angle of incidence at the core and cladding surface is just equal to critical angle, so the ray will not suffer total internal reflection. Hence beam can not propagate along the fiber.

13. **(b):** Since

$$\Rightarrow \quad R = \frac{\rho l}{A} = \frac{l}{n_i e(\mu_e + \mu_h)A} \quad \left[\because \quad \rho = \frac{1}{\sigma} = \frac{1}{n_i e(\mu_e + \mu_h)}\right]$$

$$\therefore \quad R = \frac{0.928 \times 10^{-2}}{2.5 \times 10^{19} \times 1.6 \times 10^{-19}(0.39 + 0.19) \times 10^{-6}}$$
$$= 4000\ \Omega = 4\ \text{k}\Omega$$

14. **(c)** : Maximum distance, $d = \sqrt{2hR}$
$= \sqrt{2 \times 100 \times 8 \times 10^6} = 40 \times 10^3 \text{m} = 40$ km

15. **(c) :** ac power gain is the ratio of change in output power to the change in input power.
ac power gain

$$= \frac{\text{Change in output power}}{\text{Change in intput power}} = \frac{\Delta V \Delta I_c}{\Delta V_i \Delta I_b}$$
$$= \left(\frac{\Delta V_C}{\Delta V_i}\right) \times \left(\frac{\Delta I_C}{\Delta I_B}\right) = A_V \times \beta_{AC}$$

where A_V is voltage gain and β_{AC} is ac current gain.

Also, $A_V = \beta_{AC} \times \left(\frac{R_0}{R_i}\right)$.

Given, $A_V = 50$, $R_0 = 200\ \Omega$, $R_i = 100\ \Omega$

Hence, $50 = \beta_{AC} \times \frac{200}{100}$

$\therefore$ $\beta_{AC} = 25$
Now, ac power gain $= A_V \times \beta_{AC}$
$= 50 \times 25 = 1250$

16. **(b,c):** As $I_E = I_B + I_C$
And collector current, $I_C = 10$ mA.
Since 90% of the total current passes through collector, so emitter current is

$$I_E = \frac{100}{90} \times 10 \approx 11 \text{ mA}.$$

$\therefore$ Base current $I_B = I_E - I_C = 11 - 10 = 1$ mA.

17. **(b,c):** The statements (b) and (c) are correct. The statement (a) is wrong because, the sky waves are the radiowaves of frequency 2 MHz to 30 MHz. The statement (d) is wrong because the critical frequency for F-layer of sky wave is 6 to 8 MHz.

18. **(a,b,d) :**

As $$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8 \text{ m s}^{-1}}{15 \times 10^3 \text{Hz}} = 20 \times 10^3 \text{m} = 20 \text{ km}.$$

Length of the antenna, required $l = \frac{\lambda}{4} = \frac{20}{4} = 5$ km

Audio signals are low frequency waves. They cannot be transmitted through sky waves as they are absorbed by atmosphere.

Effective power radiated by the antenna of length l, $P \propto \left(\frac{l}{\lambda}\right)^2$. If l decreases, P decreases.

19. (b,d) :

(b) As $\mu_f = \dfrac{\text{carrier frequency deviation}(\Delta\upsilon)}{\text{modulating frequency}(\upsilon_m)}$,

when $\mu_f > 1$. $\Delta\upsilon > \upsilon_m$.
Thus, there will be overlapping of sidebands in the frequency modulated wave.
This will result in loss of information.
As $\mu = A_m/A_c$, when $\mu > 1$, $A_m > A_c$,
i.e., amplitude of message signal is greater than the amplitude of carrier signal. This results in over modulation leading to excessive distortion of the transmitted modulated carrier wave.
Note : Here, μ_f is the modulation index of FM wave and μ is modulation index of AM wave.

20. (b,d) :
Bandwidth required for amplitude modulation $= 2\upsilon_m = 2(3\text{ kHz}) = 6\text{ kHz}$.
USB $= (\upsilon_c + \upsilon_m) = (1.5\text{ MHz} + 3\text{ kHz}) = 1503\text{ kHz}$
and LSB $= (\upsilon_c - \upsilon_m) = (1.5\text{ MHz} - 3\text{ kHz}) = 1497\text{ kHz}$

21. (a,c,d) :

As ripple factor, $r \propto \dfrac{1}{R_L \upsilon C}$, to reduce ripples,

(i) R_L should be increased.

(ii) input frequency (υ) should be increased.

(iii) capacitor with high capacitance (C) should be used.

22. (b,c,d):
At V_i = 1 V, it is the active region, where the transistor can be used as an amplifier.
At V_i = 0.5 V, it is the cut-off region, where the transistor can be used as a switch (off).
At V_i = 2.5 V, it is the saturation region, where the transistor can be used as a switch (on).

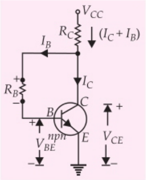
23. (b,d)

24. (b,d) : If during regulation the input voltage changes, the current through the series resistance and Zener diode also changes. Any change in the input voltage results in a change of potential difference across the series resistance without any change in voltage across the Zener diode. Since the current through Zener diode changes, resistance offered by it also changes.

25. (a,d) : Minority charge carriers are accelerated due to reverse biasing. These carriers strike the atoms causing ionisation which results in secondary electrons thereby increasing the number of charge carriers.

When doping concentration is large, there are large number of ions in depletion region. This results in a strong electric field.

26. (c) :



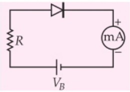
As $V_{CC} = R_C(I_C + I_B) + I_B R_B + V_{BE}$ ...(i)
and $V_{CE} = V_{CC} - (I_C + I_B)R_C \approx (V_{CC} - I_C R_C)$ (as $I_B \ll I_C$) ...(ii)
and $\beta = I_C/I_B$, ...(iii)

From (i) and (iii), $I_B = \dfrac{(V_{CC} - V_{BE})}{R_B + (\beta+1)R_C} \approx \dfrac{V_{CC}}{(R_B + \beta R_C)}$
(as $\beta >> 1$ and V_{BE} is very small).

$\therefore \quad I_B = \dfrac{10\text{ V}}{(500 \times 10^3\ \Omega) + (100 \times 500\ \Omega)} = 18.2\ \mu\text{A}$

27. (d): As $V_C = V_{CE} = V_{CC} - I_C R_C$ ...(i)
or $I_E \approx I_C = \beta I_B$
$100 \times 18.2 \times 10^{-6}\text{ A} = 1.82 \times 10^{-3}\text{ A}$
$\therefore \quad V_C = 10 - 1.82 \times 10^{-3} \times 500$ Using (i)
$= 10 - 0.91 = 9.1\text{ V}$

28. (b): Voltage drop across R,



$V_R = V_B - V_N = 5 - 0.7 = 4.3\text{ V}$
Here, $I_{\min} = 1 \times 10^{-3}$A,

$R_{\max} = \dfrac{V_R}{I_{\min}} = \dfrac{4.3}{1 \times 10^{-3}}$

$= 4.3 \times 10^3\ \Omega = 4.3\text{ k}\Omega$

29. (b): Here $I = 6\text{ mA} = 6 \times 10^{-3}$A,
$V_R = V_B - V_N = 5 - 0.7 = 4.3\text{ V}$

$\therefore \quad R = \dfrac{V_R}{I} = \dfrac{4.3}{6 \times 10^{-3}} = 717\ \Omega$

30. (a) : Boolean expression for

Circuit in (P), $Y = \overline{\overline{A \cdot B}} = A \cdot B$

Circuit in (Q), $Y = \overline{\overline{A + B}} = A + B$

Circuit in (R), $Y = \overline{A \cdot B} = \bar{A} + \bar{B}$

Circuit in (S), $Y = \overline{A + B} = \bar{A} \cdot \bar{B}$

31. (d) : Microphone converts speech signal into electrical signals. Piezo-electric senser converts pressure variation into electrical signals.

Photo-detector converts light signals into electrical signals.

Attenuation is the loss of strength of signal during its propagation through communication channel.

32. **(c) :** In AM modulation, the amplitude of the carrier signal varies in accordance with the information signal. AM signals are noisy because of electrical noisy signals significantly affect this. In FM modulation, amplitude of carrier wave is fixed while its frequency is changing. FM gives better quality transmission. It is preferred for transmission of music.

Demodulation is the process in which the original modulating voltage is recovered from the modulated wave.

33. **(b):** The modulation index determines the strength and quality of the transmitted signals.

If the modulation index is small the amount of variation in the carrier amplitude will be small. Consequently the audio signal being transmitted will not be strong.

Hence, for high modulation index is the greater degree of modulation and the audio signal reception will be clear and strong.

34. **(d) :** $R_f = \dfrac{\Delta V_f}{\Delta I_f} = \dfrac{(2-0.5)\text{ V}}{1.5\times10^{-3}\text{ A}}$

$= 10^3\ \Omega = 1\text{ k}\Omega$

35. **(a)**

36. **(5) :** Here $P = 1$ W, $V_Z = 5$ V

$V_i = 3$ V to 6 V

As $I_{z_{\max}} = \dfrac{P}{V_z} = \dfrac{1\text{ W}}{5\text{ V}} = 0.2$ A,

or $R_S = \dfrac{V_i - V_z}{I_{z_{\max}}} = \dfrac{6\text{ V} - 5\text{ V}}{0.2\text{ A}} = \dfrac{1}{0.2} = 5\ \Omega$

Hence for safe operation the value of resistance should be 5 Ω.

37. **(9) :** Here, $\upsilon_s = 4.5$ kHz, $\upsilon_c = 3.45$ MHz $= 3450$ kHz

Band width $= 2\,\upsilon_s = 2\times4.5 = 9$ kHz.

38. **(1) :** Let A_c and A_m be the amplitude of carrier waves and message signal wave.

Given,

$A_{\max} = A_c + A_m = 10$ V ...(i)

$A_{\min} = A_c - A_m = 0$ ...(ii)

On solving, (i) and (ii) we get,

or $A_c = 5$ V, $A_m = 5$ V

Modulation index, $\mu = \dfrac{A_m}{A_c} = \dfrac{5}{5} = 1$

39. **(6) :** Here, mean free path, $x = 4\times10^{-8}$ m.

Energy acquired by electron before it collides with a copper ion/atom, $W = 2.4$ eV $= 2.4\times1.6\times10^{-19}$ J.

If E is the electric field applied, then force on electron, before it collides with a copper ion/atom is

W = force × mean free path $= eEx$

$\therefore\ 2.4\times1.6\times10^{-19} = 1.6\times10^{-19}\times E\times4\times10^{-8}$

or $E = \dfrac{2.4}{4\times10^{-8}} = 6\times10^{7}\text{ V m}^{-1}$

$\therefore\ a = 6$

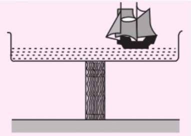
40. **(3) :** $A_V = \dfrac{V_0}{V_i} = \beta\dfrac{R_0}{R_i}$

or $R_0 = \dfrac{V_0 R_i}{V_i\beta} = \dfrac{3\times1}{0.01\times100} = 3\text{ k}\Omega$

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?
Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

Q1. A vessel filled with water is placed exactly in middle of a thin wall as shown in figure. Will the system topple if a small wooden boat carrying some weight is floated in the vessel?



– Vipin Kumar, Ghaziabad (U.P.)

Ans. The system will not topple since according to Pascal's law, the pressure on the bottom of the vessel will be same everywhere. Thus the body will still remain in equilibrium.

Q2. Assume that a hydrogen atom, initially in its ground state, absorbs a photon. In general, how many photons would you expect to be associated with the de-excitation process back to the ground state?
(a) can there be more than one photon or
(b) must there be only one photon?

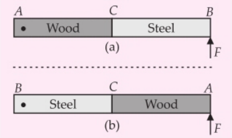
– Akhilesh (M.P.)

Ans. Since the difference between light emission and absorption is the direction of the transition (*i.e.* down for emission, up for absorption), we might be tempted to answer (b), because only one photon was required for the excitation process. However, if we look at the details of the two processes, we realize that they are not necessarily symmetrical. On absorbing a photon's energy, a hydrogen atom will jump from its ground state to an excited state, let us say from $n = 1$ to $n = 4$. This process requires a single photon of unique energy (that is, light of unique wavelength). However, in dropping back to the ground state, the atom may take one of several possible routes. For example, the atom could go from the $n = 4$ state to the $n = 3$ state, followed by a transition to the ground state ($n = 1$). This process would involve the emission of two photons. Therefore, the answer is (a). In general following the absorption of a single photon, several photons can be emitted.

Q3. There is a stick whose one half is wood and the other half is steel. It is pivoted at the wooden end and a force is applied at right angles to its length at the steel end. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why?

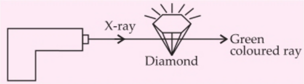
– Arumukhan, Coimbatore (Tamil Nadu)

Ans. AB is a stick whose one half (AC) is of wood and the other half (BC) is of steel. Let the stick be first pivoted at A and then at B as shown in figure (a) and figure (b) respectively. The distribution of mass about the axis of rotation is different in the two cases.



In the first case, most of the mass due to steel is farther from the axis of rotation. Obviously, the moment of inertia (I) in the first case is larger than in the second case. Since the force (F) applied and also its distance from the axis of rotation is the same in both the cases, the torque (τ) is also same in both the cases. Further, as $\tau = I\alpha$, $\alpha = \tau/I$. Since I is less in the second case than in the first, α (angular acceleration) is more in the second case than in the first case.

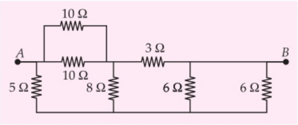
Q4. When the X-rays pass through the original diamond then, a green coloured ray is observed. Why?



– MD Faizaluddin, Hajpur (Bihar)

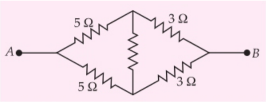
Ans. Since the critical angle for diamond is very small, therefore the X-ray photons undergo total internal reflection inside the diamond crystals. Since all colours of different wavelength undergo total internal reflection, therefore, diamond shines with an average of all the seven components (VIBGYOR) of white light, *i.e.* green.

Q5. Seven resistances are connected as shown in the figure. Find the equivalent resistance between A and B.



– Mayank

Ans. The equivalent circuit is shown in figure, which is a balanced Wheatstone bridge. Hence resistance 8 Ω is ineffective.



Total resistance between A and B is

$$= \frac{(5+3)\times(5+3)}{(5+3)+(5+3)} = 4\ \Omega$$

Q6. Why is the velocity of light in vacuum, c considered as ultimate speed limit of the particle? However linear velocity may cross this limit from equation, $v = r\omega$ (v = linear velocity, r = radius, ω = angular velocity), when ω is very-very large as close to velocity of light in vacuum.

– Madhusudan Khanal, Kalimpong

Ans. The velocity of light is 'c' in free space only. Light as per the particle theory of light is composed of photons which are massless. Any particle which has to be massless should keep on travelling with 'c' in free space. This is because mass as per the relativistic theory is

$$m(t) = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$

The numerator 'm_0' can be zero and the fraction can be consistent only if 'v', the speed of particle is equal to c. The formula $v = r\omega$ is only for classical mechanics, *i.e.*, when $v << c$ for motion of particles about a fixed point. It is not applicable on fast moving particles like photons.

Q7. Perhaps that every student may able to give atom bomb as an example of conversion of mass into energy. Then, what is the example of conversion of energy into mass?

– Madhusudan Khanal, Kalimpong

Ans. Going by your logic, *i.e.* if every student can give an atom bomb as example of conversion of mass into energy, then the example of conversion of energy to mass is that the energy required to bind together each nucleus, *i.e.* protons and neutrons is 9.315×10^8 eV. This shows that during structure formation how the formation of matter decreases the overall energy and makes the system stable. This is because an amount of energy equal to 9.315×10^8 eV has to be released whenever the neutrons and protons bond. Seeing that you have millions and millions of nucleons in the body of every student how much extra energy each one have contributed to in just their existence.

PHYSICS MUSING

SOLUTION SET-5

1. As we know that negative of potential energy gradient is force for a conservative field.

$$-\frac{dU}{dr} = F$$

It is given that $U = -\frac{ke^2}{3r^3}$

Hence, force $F = -\frac{dU}{dr} = -\frac{d}{dr}\left(\frac{ke^2}{3r^3}\right) = \frac{ke^2}{r^4}$

According to Bohr's theory this force provides the necessary centripetal force for orbital motion.

$$\therefore \quad \frac{ke^2}{r^4} = \frac{mv^2}{r} \quad \text{... (i)}$$

Also $mvr = \frac{nh}{2\pi}$ ... (ii)

Hence, $v = \frac{nh}{2\pi mr}$ ... (iii)

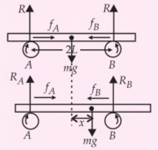
Substituting this value in Eq. (i), we get

$$\frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r^4} \quad \text{or} \quad r = \frac{4\pi^2 e^2 km}{n^2h^2}$$

Substituting this value of r in Eq. (iii), we get

$$v = \frac{n^3h^3}{8\pi^3 km^2e^2}$$

2. When the plank is situated symmetrically on the drums, the reactions on the plank from the drums will be equal and so the force of friction will be equal in magnitude but opposite in direction and hence, the plank will be in equilibrium along vertical as well as in horizontal direction.



Now if the plank is displaced by x to the right, the reaction will not be equal. For vertical equilibrium of the plank

$$R_A + R_B = mg \quad \text{... (i)}$$

And for rotation of plank, taking moment about center of mass we have

$$R_A(L + x) = R_B(L - x) \quad \text{... (ii)}$$

Solving equations (i) and (ii), we get

$$R_A = mg\left(\frac{L-x}{2L}\right) \text{ and } R_B = mg\left(\frac{L+x}{2L}\right)$$

Now as $f = \mu R$, so friction at B will be more than at A and will bring the plank back, *i.e.*, restoring force here.

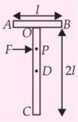
$$F = -(f_B - f_A) = -\mu(R_B - R_A) = -\mu\frac{mg}{L}x$$

As the restoring force is linear, the motion will be simple harmonic motion with force constant

$$k = \frac{\mu mg}{L}$$

So that $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{\mu g}}$.

3. It is a case of translation motion without rotation. Therefore, the force should act at the centre of mass.



Let μ is mass per unit length.

Then mass of AB, $m_1 = \mu l$ acts at O

Mass of OC, $m_2 = \mu(2l)$ acts at D, where $CD = l$.

Let $DP = x$

As P is the centre of mass, therefore,

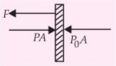
$$\mu l(l - x) = 2\mu lx$$

$$3x = l \text{ or } x = \frac{l}{3}$$

$$\therefore \quad CP = CD + DP = l + \frac{l}{3} = \frac{4l}{3}$$

4. **(b):** Volume of the gas is constant V = constant

$\therefore \quad P \propto T$



i.e. pressure will be doubled if temperature is doubled

$\therefore \quad P = 2P_0$

Now let F be the tension in the wire.

Then equilibrium of any one piston gives

$$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$$

5. **(c):** $dU = C_V dT = \left(\frac{5}{2}R\right)dT$

or $dT = \frac{2(dU)}{5R}$ ... (i)

From first law of thermodynamics

$$dU = dQ - dW$$

$$= Q - \frac{Q}{4} = \frac{3Q}{4} \quad \text{... (ii)}$$

Now molar heat capacity

$$c = \frac{dQ}{dT} = \frac{Q}{\frac{2(dU)}{5R}} = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R \text{ (using (i) \& (ii))}$$

6. **(b):** For K_α X-ray line,

$$\frac{1}{\lambda_\alpha} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R(Z-1)^2\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda_\alpha} = \frac{3}{4}R(Z-1)^2 \quad \text{... (i)}$$

With reference to given data,

$$\lambda_\alpha = 0.76 \text{ Å} = 0.76 \times 10^{-10} \text{ m}$$

$R = 1.097 \times 10^7$ m^{-1}

Putting these values in equation (i)

$$(Z-1)^2 = \frac{4}{3}\left(\frac{1}{0.76\times10^{-10}\times1.097\times10^7}\right)$$

$$\cong 1600$$

$\Rightarrow \quad Z-1 = 40 \quad \therefore \quad Z = 41$

7. **(c) :** ${}_1H^2 + {}_1H^2 \to {}_2He^4 + Q$

$\Rightarrow \quad \Delta m = m({}_2He^4) - 2m({}_1H^2)$

$\Rightarrow \quad \Delta m = 4.0024 - 2(2.0141)$

$\Rightarrow \quad \Delta m = -0.0258$ amu

Since, $Q = \Delta mc^2$

$\Rightarrow \quad Q = (0.0258)(931.5)$ MeV

$\Rightarrow \quad Q = 24$ MeV

8. **(a) :** The displacement of the particle is given by $x = A\sin(-2\omega t) + B\sin^2\omega t$

$$= -A\sin 2\omega t + \frac{B}{2}(1-\cos 2\omega t)$$

$$= -\left(A\sin 2\omega t + \frac{B}{2}\cos 2\omega t\right) + \frac{B}{2}$$

This motion represents SHM with an amplitude,

$\sqrt{A^2 + \frac{B^2}{4}}$ and mean position $\frac{B}{2}$.

9. **(d):** Frequency of the fundamental note in closed pipe is $\frac{v}{4l_1} = \upsilon_1$. Only the odd harmonics,

$3\upsilon_1$ (first overtone)

$5\upsilon_1$ (second overtone)

$7\upsilon_1$ (third overtone)

- - - - - - - - - - - - - - - -

are present

Frequency of the fundamental note in open pipe is $\frac{v}{2l_2} = \upsilon_2$. All the harmonics

$2\upsilon_2$ (second harmonic or first overtone)

$3\upsilon_2$ (third harmonic or second overtone)

$4\upsilon_2$ (fourth harmonic or third overtone)

- -

are present

Given that, $7\upsilon_1 = 3\upsilon_2$

or $\quad 7\frac{v}{4l_1} = 3\frac{v}{2l_2} \quad \therefore \quad \frac{l_1}{l_2} = \frac{7}{6}$

10. **(c) :** In the first case frequency $\omega_1 = \omega$ and in the second case frequency $\omega_2 = 2\omega$

As energy $\propto$ (frequency)^2

$$\therefore \quad \frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} = \frac{1}{4} \quad \text{or} \quad E_2 = 4E_1$$

At a Glance PHYSICS for YOU 2013

Months	Solved Papers (2013)	Practice Papers (2013 & 2014)	NEET Accelerated Learning Series/ JEE Foundation Series	NCERT Xtract	Examiner's Mind	Others
JANUARY		JEE Advanced (Full Length) NEET/PMTs, CBSE Board, JEE Advanced, Exam Prep	Dual Nature of Radiation and Matter, Atoms and Nuclei, Electronic Devices	Motion in a Plane	Some Concepts in Modern Physics	Brain Map (Semiconductor Devices)
FEBRUARY		JEE Advanced (Full Length), NEET/PMTs, CBSE Board, JEE Main, Exam Prep, NEET, 2013 MBBS		Laws of Motion	Seeking Unity in Diversity and Diversity in Unity	Brain Map (Digital Electronics and Logic Gates), Essential Formulae-Class XI, Thought Provoking Problems (SHM and Waves)
MARCH		JEE Advanced (Full Length), NEET/PMTs, JEE Main, NEET, Exam Prep		Work Energy and Power	Co-ordinates and their Importance in Physics	Brain Map (Communication Systems) Essential Formulae-Class XII, Thought Provoking Problems (Capacitors), Test Your Physics Aptitude
APRIL	CBSE Board (2013)	JEE Advanced NEET/PMTs, AIIMS, Odisha JEE, Exam Prep		System of Particles and Rotational Motion	The Laws of Conservation	Brain Map (Newton's Laws of Motion), Thought Provoking Problems (Electrostatics), Test Your Physics Aptitude
MAY	JEE Main (2013)	JEE Advanced, Target PMTs, AIIMS, Exam Prep		Gravitat-ion	The Laws of Conservation	Brain Map (Electrostatics), Thought Provoking Problems (Kinematics), Challenging Problems, Test Your Physics Aptitude
JUNE	WB JEE (2013), Kerala PET (2013), NEET (2013)	CBSE Board Chapterwise (Series-1), Exam Prep		Mechanical Properties of Solids	Forces, Fields and Co-ordinates	Brain Map (System of Particles and Rotational Motion), Thought Provoking Problems (Electrostatics), Concept Based Problem for JEE, Challenging Problems
JULY	JEE Advanced (2013), Karnataka CET 2013, AMU Medical 2013	Exam Prep, CBSE Board Chapterwise (Series-2)		Mechanical Properties of Fluids		Brain Map (Current Electricity), Thought Provoking Problems (Current Electricity), Challenging Problems, Test Your Physics Aptitude
AUGUST		JEE Main and Advanced, CBSE Board Chapterwise (Series-3), Examp Prep, Target PMTs	Unit and Measurement, Motion in One Dimension, Projectile Motion	Thermal Properties of Matter		Brain Map (Gravitation), Thought Provoking Problems (Dynamics), Challenging Problems, Physics Musing Problems
SEPTEMBER		CBSE Board Chapterwise (Series-4), Target PMTs, Exam Prep	Newton's Laws of Motion, Work, Energy and Power	Thermo-dynamics	Units and Measurements, Kinematics (Class XI), Electrostatics and Current Electricity (Class XII)	Brain Map (Magnetic Effects of Current), Physics Musing Problems and Solutions
OCTOBER		CBSE Board Chapterwise (Series-5), Exam Prep, Target PMTs,	Rotational Motion, Gravitation	Kinetic Theory	Work, Energy and Power Rotational Motion (Class XI), Magnetic Effects of Current, Electromagnetic Induction and AC (Class XII)	Brain Map (Mechanical Properties of Solids and Fluids), Thought Provoking Problems (EMI), Physics Musing Problems and Solutions
NOVEMBER		CBSE Board Chapterwise (Series-6), Exam Prep	Properties of Solids and Liquids, Thermodynamics	Oscillations	Gravitation, Mechanical Properties of Solids (Class XI), Ray Optics and Optical Instruments, Wave Optics (Class XII)	Brain Map (Electromagnetic Induction and AC) Physics Musing Problems and Solutions, You Asked, We Answered
DECEMBER		CBSE Board, Chapterwise (Series-7), Exam Prep	Kinetic Theory of Gases, Oscillations, Waves	Waves	Mechanical Properties of Fluids, Thermal Properties of Matter, Thermodynamics (Class XI), Dual Nature of Radiation and Matter, Atoms, Nuclei (Class XII)	Brain Map (Heat and Thermo-dynamics), Thought Provoking Problems (Optics), Physics Musing Problems and Solutions, You Asked, We Answered